Intangible Capital and Ramsey Capital Taxation

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The standard analysis of optimal fiscal policy in the neoclassical growth model, e.g. Chamley (1986) and Judd (1985), aggregates different types of assets into a unique capital good and all sorts of capital taxes into a unique capital tax. There, the optimal capital tax rate is very high in the short-run and zero in the long-run and, inevitably, time-inconsistent. This paper shows that this classic result does not hold in a more disaggregated framework. As proposed in McGrattan and Prescott (2005), we consider an economy with corporate and dividend taxes, where firms invest in both tangible and intangible assets. If corporate taxes are high, firms can avoid current taxation by investing in intangible assets that can be expensed. If dividend taxes are temporarily high, firms can defer dividend payments. We show that the Ramsey plan is characterized by zero corporate taxes every period (including the initial one) and constant dividend tax rates, so the Ramsey capital taxation is time-consistent.
1. Introduction

Since the seminal contributions of Chamley (1986) and Judd (1985), and going back to Kydland and Prescott (1977), we know that the optimal capital income taxation with commitment implies an asymmetry in the fiscal treatment of current versus future capital income that renders the Ramsey plan time inconsistent.\footnote{The time inconsistency of the Ramsey solution is well documented and has generated a vast literature on identifying the best time consistent policy when the ability of the fiscal authority to commit is restricted in different ways. See, among others, Phelan and Stacchetti (2001), Klein, Krusell and Ríos-Rull (2004), Domínguez (2006).} We show that this time inconsistency result is not robust to the explicit consideration of a corporate sector that invests in both tangible and intangible assets, as proposed in McGrattan and Prescott (2005). They proposed such a model in order to understand the implications of changes in corporate income and dividend taxation on the valuation of the stock market. Their analysis was a positive one. Our contribution is to incorporate the normative view into the picture, and show that it leads to very drastic changes in the way we think of the design of capital income taxation.

The classical theoretical framework aggregates different type of assets into a unique taxable capital good. However, the structure of capital income taxation is far more complex than this. Corporations report their income, which is directly taxed, then decide how much dividends to distribute to their shareholders, and these dividends become taxable personal income, together with the returns on other financial assets. In applied analysis the empirical counterpart of the capital income tax is obtained by adding up the revenues generated from the taxation of corporate income and the taxation of the return on different financial assets (including capital gains), divided by some measure of aggregate capital income. This measure is usually referred to as the effective tax rate on capital income.

In this paper, we examine optimal fiscal policy in a framework that explicitly takes into account the disaggregated structure of capital income taxation mentioned above. We show that the explicit consideration of corporate income and dividend taxation generates an asymmetry in the fiscal treatment of tangible and intangible assets. As a result, a benevolent
fiscal authority would choose to remove this asymmetry, and that necessarily implies zero corporate income taxes every period, including the initial one. In addition, such a fiscal authority would choose not to distort the timing of dividend payments, resulting in constant dividend tax rates. These constant dividend tax rates are non-distortionary and, thus, set as high as possible. Therefore, the optimal corporate and dividend tax rates are constant and, as a result, time-consistent.

This result arises when we restrict ourselves to policies converging to a steady state. In the more general case the optimal policy may imply corporate income taxes converging to one from below, while dividend taxes converge to minus infinity. We view this outcome as empirically irrelevant. Moreover, a basic extension introducing managerial effort rules out this possibility. The second part of the paper extends our analysis to incorporate this feature. Managerial or entrepreneurial effort can be seen as time devoted by worker-owners of the corporate firm to the transformation of resources into new capital, which we assume necessary to ensure the effectiveness of investment. Examples of entrepreneurial effort are searching for new investment opportunities, managing research projects, etc. See Albanesi (2005), McGrattan and Prescott (2006), and Zhu (1995) for models with entrepreneurial effort. In particular, McGrattan and Prescott (2006) have found that incorporating this effort is crucial in order to understand the movements in hours and productivity in the 1990s.

The introduction of managerial effort in our analysis is also important because now the taxation of dividends is non-trivial, in the sense that it is no longer non-distortionary. We show that our basic results are robust to this extension. Moreover, now the unconstrained Ramsey policy implies zero corporate income taxes at all dates and a constant non-confiscatory dividend tax.

Our results are in sharp contrast with existing wisdom about the taxation of capital income. In the traditional literature the optimality of zero capital taxes is always a long-run result, but never at the initial date. The optimal capital tax at the initial date (and in the short-run) is always set as large as possible due to the capital levy problem. Moreover, these initial high capital taxes have been shown to be quantitative important. Chari, Christiano and Kehoe (1994) showed that about 80% of the welfare gains from switching to the standard Ramsey system come from the high taxes on the initial capital income. In our
analysis, however, we find that these high initial capital taxes are not optimal since they would generate an inefficient unequal treatment of tangible and intangible capital (which would boost intangible investment, reduce tangible investment, and would not entail a sizable tax collection).

There is substantial empirical evidence that corporations do react to changes in the fiscal treatment of corporate income, fiscal deductions or dividends. See Auerbach (2002) for an excellent account of the theoretical and empirical literature on corporate taxation and its implications for corporations’ financial policies. Turnovsky (1990) provides a detailed analysis of the dynamic macroeconomic implications of capital income tax changes. Our work differs from theirs in that we analyze the optimal taxation following the Ramsey approach and that we allow for investment in intangible assets.

A related result has been recently shown in Abel (2006). He shows that immediate expensing of depreciation allowances renders constant capital income taxation non-distortionary. As in our analysis, there is no incentive for an initial capital levy and the optimal taxation of capital income is time consistent.

The rest of the paper is organized as follows. Section 2 describes the basic model, Section 3 characterizes the optimal corporate and dividend taxation, Section 4 extends the analysis by introducing managerial effort and Section 5 concludes. The proofs of the propositions are included in the Appendix.

2. The Basic Model

We follow the same formulation as in McGrattan and Prescott (2005). The economy is composed of a household sector, a corporate sector and the government.

The Household Sector
We represent households’ preferences by a utility function defined as the discounted infinite stream of the instantaneous flow of utility derived from consumption, \(c_t\), and leisure, \(\ell_t\), so that preferences are defined as

\[
\max_{c_t, \ell_t} \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)
\]
\[
\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t),
\]

where \( \beta \in (0,1) \), and \( u(,..) \) is strictly increasing, strictly concave and continuously differentiable in both arguments.

Households are worker-owners of the firms in the corporate sector. They own and trade shares in the ownership of corporations in a competitive market, for which they receive every period a dividend \( d_t \) per share. They rent labor services to corporations in exchange of a competitive wage \( w_t \). We normalize time available for the household to 1, so that time devoted to work is \( n_t = 1 - \ell_t \).

The households’ sources of income are labor income, dividends from corporations, together with the interest payments on government bonds. Total income is used to buy consumption goods, new shares of corporations and new government bonds. Hence the consumers’ sequential budget constraint of period \( t \) is given by

\[
c_t + v_t(s_{t+1} - s_t) + (b_{t+1} - b_t) \leq (1 - \tau_t^n) w_t n_t + (1 - \tau_t^d) d_t s_t + (1 + \tilde{r}_t^b) b_t,
\]

where \( v_t \) is the non-negative time \( t \) price of one share, \( s_t \leq 1 \) is the number of shares owned by the household at time \( t \), \( \tau_t^n \) is the tax rate on labor income, \( \tau_t^d \) is the tax rate on dividend payments, \( b_t \) denotes the household’s holdings of government bonds and \( \tilde{r}_t^b \) is the interest rate on the bond. Since one could not force households to own an asset with negative value, we assume that the value of corporate shares, \( v_t \), is always non-negative. This is equivalent to assuming limited liability. We assume that \( b_t \geq -B \), and \( s_t \geq -S \). Notice that short sales of equity or government bonds are allowed. We assume that \( B \) and \( S \) are positive constants big enough not to bind in equilibrium, in order to prevent Ponzi schemes. Since we focus on aggregate behaviour, it is the case that \( s_t = 1 \) in equilibrium (see the definition of equilibrium), i.e. the aggregate consumer owns all the shares.

The following first order conditions are necessary for a solution to the household’s
maximization problem:

\[
\begin{align*}
[c_t] & \quad \beta^t u_{c,t} - p_t = 0, \\
[n_t] & \quad \beta^t u_{n,t} + p_t (1 - \tau^t) w_t = 0, \\
[s_{t+1}] & \quad -p_t v_t + p_{t+1} [(1 - \tau^t) d_{t+1} + v_{t+1}] = 0, \\
[b_{t+1}] & \quad -p_t + p_{t+1} [1 + \tilde{r}^t v_{t+1}] = 0,
\end{align*}
\]

where \( p_t \) denotes the Lagrange multiplier on the budget constraint.

The Corporate Sector

The corporate sector is composed of a continuum (measure 1) of identical firms operating in a competitive environment. Each one of them produces output with a constant returns to scale production technology \( y_t = f(k_{m,t}, k_{u,t}, n_t) \). The inputs in the production function are hours worked, \( n_t \), physical (or tangible) assets, \( k_{m,t} \), which are measured, and intangible assets, \( k_{u,t} \), which are unmeasured. These assets depreciate respectively at the rates \( \delta_m \) and \( \delta_u \), both positive but smaller than unity and potentially different.

The corporations’ objective function is to maximize the net present value of the infinite stream of dividends, which is equal to the initial value of the firm. Corporate income is defined as the value added net of depreciation of tangible assets, labor income and investment in intangible assets, \( x_{u,t} \), and is taxed at a rate \( \tau^c_t \). After-tax corporate income is either invested in tangible capital, \( x_{m,t} \), or distributed as dividends. Therefore, dividends paid by corporations are given by

\[
d_t = (1 - \tau^c_t) \left[ f(k_{m,t}, k_{u,t}, n_t) - x_{u,t} - w_t n_t \right] + \tau^c_t \delta_m k_{m,t} - x_{m,t},
\]

where

\[
\begin{align*}
x_{m,t} &= k_{m,t+1} - (1 - \delta_m) k_{m,t}, \\
x_{u,t} &= k_{u,t+1} - (1 - \delta_u) k_{u,t}.
\end{align*}
\]
Hence, the net present value of dividend payments (initial value of the firm) equals

\[
\sum_{t=0}^{\infty} p_t (1 - \tau^d_t) d_t = \sum_{t=0}^{\infty} p_t (1 - \tau^d_t) \left\{ (1 - \tau^d_t) \left[ f(k_{m,t}, k_{u,t}, n_t) - \delta_m k_{m,t} - w_t n_t - k_{u,t+1} + (1 - \delta_u) k_{u,t} \right] - k_{m,t+1} + k_{m,t} \right\}.
\]

Here it is important to notice the role played by the assumption that intangible assets are unmeasured. Corporations can hide present corporate income from tax authorities via investment in intangible assets, generating higher future corporate income. We think of a corporation devoting productive resources (or units of the final good) to activities such as advertisement, building a distribution network, developing new ideas, etc. As a consequence of such activities measured value added will be smaller, \( f(k_{m,t}, k_{u,t}, n_t) - x_{u,t} \).

Notice also we do not place any constraint on the sign of dividends. We interpret negative dividends as the corporate sector issuing more equity to finance investment.

Any solution to the corporate sector maximization problem must satisfy the following first order conditions:

\[
\begin{align*}
[d_t] (1 - \tau^d_t) p_t - \mu_t &= 0, \\
[k_{m,t+1}] - \mu_t + \mu_{t+1} \left[ 1 + (1 - \tau^e_{t+1}) (f_{m,t+1} - \delta_m) \right] &= 0, \\
[k_{u,t+1}] - (1 - \tau^e_t) \mu_t + (1 - \tau^e_{t+1}) \mu_{t+1} \left[ 1 + f_{u,t+1} - \delta_u \right] &= 0, \\
[n_t] f_{u,t} - w_t &= 0;
\end{align*}
\]

and the transversality conditions for tangible and intangible capital, which respectively are

\[
\lim_{t \to \infty} (1 - \tau^d_t) p_t k_{m,t+1} = 0 \quad \text{and} \quad \lim_{t \to \infty} (1 - \tau^d_t) (1 - \tau^e_t) p_t k_{u,t+1} = 0. \tag{4}
\]

The optimality conditions for tangible and intangible capital can be rewritten as

\[
\begin{align*}
(1 - \tau^d_t) p_t &= (1 - \tau^d_{t+1}) p_{t+1} \left[ 1 + (1 - \tau^e_{t+1}) (f_{m,t+1} - \delta_m) \right], \tag{5} \\
(1 - \tau^d_t)(1 - \tau^e_t) p_t &= (1 - \tau^d_{t+1})(1 - \tau^e_{t+1}) p_{t+1} \left[ 1 + f_{u,t+1} - \delta_u \right]. \tag{6}
\end{align*}
\]

Now, using the first order conditions (5)-(6), the transversality conditions (4), and the fact
that the function \( f \) displays constant returns to scale, the initial value of the firm becomes

\[
\sum_{t=0}^{\infty} (1 - \tau_t^d) p_t d_t = (1 - \tau_0^d) u_{c,0} \left\{ (1 - \tau_0^c) \left[ f_{m,0} k_{m,0} + f_{u,0} k_{u,0} + (1 - \delta) k_{u,0} - \delta k_{m,0} \right] + k_{m,0} \right\}. \quad (7)
\]

**The Government**

The government collects tax revenues in order to finance an exogenously given stream of government consumption, denoted by \( \{ g_t \}_{t=0}^{\infty} \), and issues one-period government bonds. We assume that government consumption is unproductive and is not valued by households. Tax revenues are collected through taxation on labor income, \( \tau_t^n \), on corporate income, \( \tau_t^c \) and on dividend payments \( \tau_t^d \). We assume that tax rates can be positive or negative but are bounded above by \( \tau_{\text{max}} < 1 \), which could be very close to one, and that \( b_t \leq B \) in order to rule out Ponzi schemes. Hence, the government sequential budget constraint is given by:

\[
\tau_t^n w_t n_t + \tau_t^c \left[ f(k_{m,t}, k_{u,t}, n_t) - x_{u,t} - w_t n_t - \delta_m k_{m,t} \right] + \tau_t^d d_t s_t + b_{t+1} \geq g_t + (1 + \tau_t^b) b_t. \quad (8)
\]

**Definition of a Competitive Equilibrium**

Given a fiscal policy \( \{ \tau_t^n, \tau_t^c, \tau_t^d, g_t \}_{t=0}^{\infty} \), a competitive equilibrium is a sequence of households’ allocations \( \{ \hat{c}_t, \hat{f}_t, b_{t+1}, s_{t+1} \}_{t=0}^{\infty} \), firms’ production and distributions plans \( \{ \hat{d}_t, \hat{k}_{m,t+1}, \hat{k}_{u,t+1}, \hat{n}_t \}_{t=0}^{\infty} \), and prices \( \{ \hat{w}_t, \hat{r}_t^b, \hat{p}_t, \hat{v}_t \}_{t=0}^{\infty} \) such that:

(i) Given prices and policies, the households’ allocation maximizes welfare (1) subject to the budget constraint (2), \( \hat{n}_t + \hat{\ell}_t \leq 1, \hat{b}_t \geq -B, \text{ and } \hat{s}_t \geq -S \), for some initial conditions on \( b_0 \) and \( s_0 = 1 \).
(ii) Given prices and policies, the firms’ production and distribution plan maximizes the discounted sum of dividends \( \sum_{t=0}^{\infty} (1 - \tau_t^d) \hat{p}_t \hat{d}_t \) for some initial, \( k_{m,0} \), \( k_{a,0} \), and transversality conditions (4).

(iii) The labor market is cleared \((\hat{\ell}_t + \hat{n}_t = 1)\), the equity market is cleared \((\hat{s}_t = 1)\), and the government budget constraint (8) is satisfied. Feasibility requires

\[ \hat{c}_t + \hat{x}_{m,t} + \hat{x}_{u,t} + g_t \leq f(k_{m,t}, k_{u,t}, \hat{n}_t). \]

3. The Ramsey Problem

We now turn to the government problem. To do that, we assume that there is a commitment technology that allows all future governments to commit to the sequence of taxes announced by the government at date 0. We also assume that \( \hat{r}_b^h \) is given, so that the initial government commits to honor debt payments.

To set up the government’s optimization problem, we follow the primal approach. First, we find the Implementability Condition (IC) by adding up the budget constraint (2) over time and using the optimality conditions \([c_t], [n_t], [s_{t+1}] \) and \([b_{t+1}]\), which yield

\[ \sum_{t=0}^{\infty} \beta^t \left[ c_t u_{c,t} + n_t u_{n,t} \right] = u_{c,0} \left[ 1 + \hat{r}_b^h \right] b_0 + \sum_{t=0}^{\infty} (1 - \tau_t^d) p_t d_t \]

\[ = u_{c,0} \left[ 1 + \hat{r}_b^h \right] b_0 + (1 - \tau_0^d) \left( (1 - \tau_0^d) \left( f_{m,0} k_{m,0} + f_{u,0} k_{u,0} + (1 - \delta_u) k_{u,0} - \delta_m k_{m,0} \right) + k_{m,0} \right). \]

Notice this is the standard (IC), where the right hand side expression is the value of initial wealth, which is composed by the initial bond holdings and the value of the households’ ownership of the corporate sector.

Then, the formulation of the Ramsey problem is to maximize households’ welfare subject to feasibility and this (IC), i.e.:
max \quad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)

s.t. \quad c_t + k_{m,t+1} - (1 - \delta_m)k_{m,t} + k_{m,t+1} - (1 - \delta_u)k_{u,t} + g_t \leq f(k_{m,t}, k_{u,t}, n_t)

\sum_{t=0}^{\infty} \beta^t \left[ c_t u_{c,t} + n_t u_{n,t} \right] =

u_{c,0} \left[ 1 + \tilde{\tau}_0^b \right] p_0 + (1 - \tau_0^d) \left\{ (1 - \tau_0^d) \left[ f_{m,0}k_{m,0} + f_{u,0}k_{u,0} + (1 - \delta_u)k_{u,0} - \delta_m k_{m,0} \right] + k_{m,0} \right\]

k_{m,0}, k_{u,0}, \tilde{\tau}_0^b \text{ given.}

The first order conditions for this problem at time \( t > 0 \) are

\begin{align*}
[c_t] & \quad u_{c,t} \left( 1 + \lambda \right) + \lambda \left( u_{c,t} c_t + u_{n,t} n_t \right) = \phi_t, \\
[n_t] & \quad u_{n,t} \left( 1 + \lambda \right) + \lambda \left( u_{c,t} c_t + u_{n,t} n_t \right) = -f_{c,t} \phi_t, \\
k_{m,t+1} & \quad \phi_t = \beta \phi_{t+1} \left( 1 + f_{m,t+1} - \delta_m \right), \\
k_{u,t+1} & \quad \phi_t = \beta \phi_{t+1} \left( 1 + f_{u,t+1} - \delta_u \right),
\end{align*}

\hspace{1cm} (9)

\hspace{1cm} (10)

where \( \beta \phi_t \) and \( \lambda \) are the Lagrange multipliers on the resource constraint and the implementability condition (IC), respectively. Clearly, equations (9) and (10) imply that \( f_{m,t} - \delta_m = f_{u,t} - \delta_u \), \( \forall t \geq 1 \).

First we will focus on a subset of Ramsey policies characterized by convergence to a steady state policy. Therefore, in the following result we explicitly restrict the set of policies we look at.

**Proposition 1.** If the Ramsey policy converges to a steady state in finite periods, then it is characterized by the following:

(i) The optimal corporate tax rate is equal to zero at all dates \( t \geq 0 \).

(ii) Let \( u(c, \ell) \) be separable in \( c \) and \( \ell \), and homothetic in \( c \). Then, the optimal dividend tax is constant over time and equal to \( \tau^{max} \) at all dates \( t \geq 0 \).

Proof. See the Appendix.
We first explain the results and then discuss the issue of Ramsey policies in a broader set (policies that might not converge in steady state). First, since any solution to the Ramsey problem must satisfy the equality of the net returns to tangible and intangible capital and firms can choose distributions in all periods \( t \geq 0 \), then corporate taxes must be always zero, even at the initial date. In other words, a positive corporate tax is not efficient because it taxes tangible but it cannot tax intangible investment.\(^2\)

Second, we obtain that the dividend tax should be set as high as possible. As it is clear from the IC, the initial dividend tax is non-distortionary and should be set as high as possible. Moreover, this is true not only at the initial period but always. As can be seen from the firm’s first order conditions (5) and (6), the dividend tax rate at period \( t \) depends on the tax rate at period \( t-1 \). Moreover, for homothetic and separable utility functions, Proposition 1 finds that dividend taxes should be constant. The intuition for this result is that, since firms can choose the timing of distributions, it is optimal not to distort the timing. For other types of utility function, dividend taxes might change over time but these changes are quantitatively small.

Notice that constant dividend taxes have no impact on the firm’s allocation of real resources, but they directly determine the value of the firm, see McGrattan and Prescott (2005). Inspection of (2) indicates that dividend taxation, not only in period 0 but always, is non-distortionary from the households’ perspective as well. In the next section we introduce and analyze a model with entrepreneurial effort, where dividend taxes will be distortionary.

In order to characterize the optimal policy, we have constrained ourselves to the subset of policies that converge to a steady state in finite periods. This assumption is required for the following reason. From the firm’s FOC, combining (5) and (6), we obtain that

\(^2\) Notice that the alternative of a 100% constant corporate tax is not optimal, since then dividends would be negative for any positive level of investment in tangible capital, so eventually the corporate sector disappears.
\[ 1 - \tau_t^c = \frac{(1 - \tau_t^c)(1 + r_{t+1})}{1 + (1 - \tau_{t+1}^c)r_{t+1}} , \forall t \geq 0, \tag{11} \]

where we have denoted by \( r_t \) the real rate of return of both types of capital (which is equalized for \( t \geq 1 \)). This expression immediately shows that if the corporate income tax is constant after some point in time, then it has to be zero. Not only that, working backwards it must zero always, including the initial period.

However, if we do not constrain taxes to converge to a steady state we might identify a different optimal policy. Expression (11) shows that if corporate taxes are not zero in the initial period, the outcome would be an ever-lasting transition in taxes (even though the allocation, and therefore the return of both types of capital, is constant in finite periods). If \( \tau^{\text{max}} \) is very close to 1, this transition is characterized by corporate and dividend taxes approaching 1 and -\( \infty \), respectively.\(^3\) However, this result is not robust to small realistic changes in the model (for example, allowing for managerial effort). Thus in the next section we incorporate this feature in order to rule out this type of knife-edge equilibrium.

### 4. Introducing Managerial Effort and Sweat Equity

In this section we incorporate managerial effort in the model. In order to do that, we follow McGrattan and Prescott (2006). They study the movements in hours and productivity in the 1990s and show the importance of intangible investment in expensed and sweat equity. According to their environment expensed investment in intangible assets increases future profits but is treated as an operating expense, and sweat equity is financed by workers-owners of the firm who spend hours in their business building equity. In the previous section we had an economy with expensed investment. Here we present a version of McGrattan-Prescott’s model, incorporating both expensed and sweat investment.

\(^3\) From a theoretical perspective this type of equilibrium cannot be ruled out. It satisfies all the first order conditions (the transversality conditions are satisfied for this type of equilibria) and it might generate higher welfare than the policy considered in Proposition 1.
We assume that households, as workers-owners of the firm, will devote some time to work \( n_t \) and some time or effort \( n_{2t} = 1 - \ell - n_t \) to manage investment projects. For this second activity, they receive no wage but the value of their firm increases. We assume this management time is necessary in order to ensure the transformation of resources into new capital. In other words, we assume that the production of both tangible and intangible capital requires investment (measured in units of the final good), \( x_{m,t} \) and \( x_{u,t} \), respectively, and managerial effort \( n_{2t} \), that is

\[
I^m(x_{m,t}, n_{2,t}) = k_{m,t+1} - (1 - \delta_m)k_{m,t}, \tag{12}
\]

\[
I^u(x_{u,t}, n_{2,t}) = k_{u,t+1} - (1 - \delta_u)k_{u,t}. \tag{13}
\]

The functions \( I^j(., .), j=m,u, \) are strictly increasing and concave in both arguments. We assume that a given management time is used in the production of both types of investment. Our results will also hold if we assume that separate work effort is needed for each type of investment.

This feature changes the optimization problem of the households as follows. They now maximize

\[
\sum_{j=0}^{\infty} \beta^j u(c_t, 1 - n_t - n_{2t}),
\]

subject to the budget constraint

\[
c_t + v_t (s_{t+1} - s_t) + (b_{t+1} - b_t) = \left(1 - \tau_t^m\right) w_t n_t + (1 + \tau_t^b) b_t + \left(1 - \tau_t^u\right) \left[ r_{m,t} k_{m,t} + r_{u,t} k_{u,t} - x_{u,t} \right] + \tau_t^u \delta_m k_{m,t} - x_{m,t} \]

and equations (12) and (13). Here \( r_{m,t} \) and \( r_{u,t} \) are the rental rates of tangible and intangible capital, which are equal to their respective marginal products \( f_{m,t} \) and \( f_{u,t} \). We change notation just to make clear that the individual, as a worker, behaves competitively in the labor market but, as a manager, internalizes the effect of effort on the value of the firm.
Any solution must satisfy the following first order conditions

\[
\begin{align*}
[n_{t}]= & \quad \beta' u_{n_{t},t} + \chi^{u}_{t} I_{n_{t},t}^{u} + \chi^{m}_{t} I_{n_{t},t}^{m} = 0, \\
[x_{m,t}]= & \quad \chi^{u}_{t} I_{x,t}^{u}-(1-\tau_{t}^{d}) p_{t} s_{t} = 0, \\
[x_{u,t}]= & \quad \chi^{m}_{t} I_{x,t}^{m}-(1-\tau_{t}^{v}) (1-\tau_{t}^{u}) p_{t} s_{t} = 0, \\
[k_{m_{t+1}}]= & \quad -\chi^{u}_{t} + (1-\tau_{t+1}^{d}) p_{t+1} \left[ (1-\tau_{t+1}^{v}) r_{m_{t+1}}^{t} + \tau_{t+1}^{v} \delta_{m} \right] s_{t+1} + (1-\delta_{m}) \chi^{m}_{t+1} = 0, \\
[k_{u_{t+1}}]= & \quad -\chi^{u}_{t} + (1-\tau_{t+1}^{d})(1-\tau_{t+1}^{u}) p_{t+1} r_{u_{t+1}}^{t} s_{t+1} + (1-\delta_{u}) \chi^{u}_{t+1} = 0,
\end{align*}
\]

where \(\chi^{u}_{t}\) and \(\chi^{m}_{t}\) are the Lagrange multipliers on equations (12) and (13), respectively.

The remaining optimality conditions are just as before. Combining them and using that \(s_{t} = 1\) in equilibrium, we obtain

\[
\begin{align*}
\left(1-\tau_{t}^{d}\right) p_{t} = & \left(1-\tau_{t+1}^{d}\right) p_{t+1} \left[ (1-\tau_{t+1}^{v}) r_{m_{t+1}}^{t} + \tau_{t+1}^{v} \delta_{m} + (1-\delta_{m}) (I_{x,t+1}^{m})^{-1} \right] I_{x,t}^{u}, \\
\left(1-\tau_{t}^{v}\right) p_{t} = & \left(1-\tau_{t+1}^{v}\right) \left(1-\tau_{t+1}^{u}\right) p_{t+1} \left[ r_{u_{t+1}}^{t} + (1-\delta_{u}) (I_{x,t+1}^{u})^{-1} \right] I_{x,t}^{u},
\end{align*}
\]

and

\[
-\beta' u_{n_{t},t} = \left(1-\tau_{t}^{d}\right)(1-\tau_{t}^{v}) (I_{x,t}^{u})^{-1} I_{n_{t},t}^{u} + \left(I_{x,t}^{m}ight)^{-1} I_{n_{t},t}^{m} p_{t}.
\]

Note that, whenever \(I_{x,t}^{m} = I_{x,t}^{u} = 1\), equations (14) and (15) are simply equations (5) and (6).

Moreover, since the return to each unit of time must be the same, it must be satisfied that

\[
-\omega_{n_{t},t} = \left(1-\tau_{t}^{d}\right)(1-\tau_{t}^{v}) (I_{x,t}^{u})^{-1} I_{n_{t},t}^{u} + \left(I_{x,t}^{m}ight)^{-1} I_{n_{t},t}^{m} u_{e_{t},t} = (1-\tau_{t}^{u}) w_{t} u_{e_{t},t},
\]

This equation is very important because of two reasons. First, combining this equation with (15), it is clear that if the allocation is constant, then the policy must be constant as well. Second, this equation shows that dividend and corporate taxes are distortionary even in period 0. This equation allows us to write the initial value of the firm as
\[ \sum_{j=0}^{\infty} (1-\tau_j^d) p_{ij} = - \left( \frac{u_{x,0}^u}{u_{c,0}^u} \right) (I_{x,0}^u)^{-1} I_{x,0}^u u_{c,0}^u \left[ f_{m,0} k_{m,0} + f_{u,0} k_{u,0} + (1-\delta_u) k_{u,0} - \delta_u k_{m,0} \right] \\
+ (1-\tau_0^d) u_{c,0}^u \left[ k_{m,0} - (I_{x,0}^m)^{-1} I_{x,0}^m \left( I_{x,0}^u \right)^{-1} I_{x,0}^u k_{m,0} \right], \]

and the implementability constraint \((IC')\) as

\[ \sum_{j=0}^{\infty} \beta^j \left[ c_{u,j} u_{c,j} + n_{11} u_{n,1} \right] = u_{c,0}^u [1 + \delta_h u_{c,0}^u] - \left( \frac{u_{x,0}^u}{u_{c,0}^u} \right) (I_{x,0}^u)^{-1} I_{x,0}^u u_{c,0}^u \left[ f_{m,0} k_{m,0} + f_{u,0} k_{u,0} + (1-\delta_u) k_{u,0} - \delta_u k_{m,0} \right] \\
+ (1-\tau_0^d) u_{c,0}^u \left[ k_{m,0} - (I_{x,0}^m)^{-1} I_{x,0}^m \left( I_{x,0}^u \right)^{-1} I_{x,0}^u k_{m,0} \right]. \]

The government problem must be also modified in order to take into account managerial effort, the new implementability constraint and equations (12) and (13). We obtain the following first order conditions:

\[ \begin{bmatrix} n_{2,t} \\ x_{m,t} \\ x_{u,t} \\ k_{m,t+1} \\ k_{u,t+1} \end{bmatrix} \begin{bmatrix} \beta^t u_{n,2,t} + \chi_t^{Gu} I_{n,2,t} + \chi_t^{Gm} I_{m,2,t} = 0, \\
\chi_t^{Gm} I_{x,t} + \beta^t \phi_1 = 0, \\
\chi_t^{Gu} I_{x,t} + \beta^t \phi_1 = 0, \\
-\chi_t^{Gm} + \beta^{t+1} \phi_1 f_{m,t+1} + (1-\delta_m) \chi_t^{Gm} = 0, \\
-\chi_t^{Gu} + \beta^{t+1} \phi_1 f_{u,t+1} + (1-\delta_u) \chi_t^{Gu} = 0. \end{bmatrix} \]

Note that the first order condition for effort takes a different form at date 0. The optimality conditions can be rewritten as

\[ \phi_1 = \beta \phi_1 \left( f_{m,t+1} + (1-\delta_m) (I_{x,t+1}^m)^{-1} I_{x,t+1}^m \right), \]

\[ \phi_1 = \beta \phi_1 \left( f_{u,t+1} + (1-\delta_u) (I_{x,t+1}^u)^{-1} I_{x,t+1}^u \right), \]

and

\[ -\beta^t u_{n,2,t} = \left( (I_{x,t}^u)^{-1} I_{x,t+1}^u + (I_{x,t}^m)^{-1} I_{x,t+1}^m \right) \phi_1. \]

Now we can extend our results from the previous section to this setup with
expensed and sweat equity:

**Proposition 2.** If there exists a steady state Ramsey allocation, then the Ramsey policy is characterized by the following:

(i) The optimal corporate tax rate is equal to zero at all dates $t \geq 0$.

(ii) Let $u(c, \ell)$ be separable in $c$ and $\ell$, and homothetic in $c$. Then, the optimal dividend tax is equal to $\theta \leq \tau^{\text{max}}$ at all dates $t \geq 0$.

Proof. See the Appendix.

Therefore, as in the previous section, we find that corporate taxes should be set equal to zero from date 0 on. The difference is that now there is no alternative Ramsey policy with time varying dividend and corporate income taxes. Moreover, in this setup with managerial effort, in which dividend taxes are distortionary, we again find that the optimal dividend taxes should be constant. Furthermore, they should not be set at confiscatory rates. Otherwise, managers would not provide optimal levels of effort into their businesses.

In McGrattan and Prescott (2006), they considered that the managers’ effort is partly rewarded through a wage compensation (which is lower than the market wage). We believe our results would be robust to this extension. The main reason is that the two driving forces of our results would still be present in that scenario: the asymmetric fiscal treatment of both types of capital and the distortionary effect of time-variant dividend taxes compared to constant dividend taxes.
5. Conclusions

In this paper we have investigated the properties of optimal taxes in an environment that explicitly considers a corporate sector that invests in both tangible and intangible capital. We have derived an important implication that challenges conventional wisdom: the classical result of the time inconsistency of Ramsey capital taxes does not hold in such an environment.

This result comes from two findings. First, corporate income taxes should always be zero to eliminate the asymmetric fiscal treatment of tangible and intangible capital. Second, dividend taxes should always be constant to eliminate distortions in the timing of dividend payments and, thus, in investment. A capital levy is not feasible due to the existence of unobservable intangible investment (that can shrink at will the corporate income tax base) and the ability by corporations of differing dividend payments. Since capital levies are not feasible, the Ramsey capital taxes are time consistent.

This finding opens up several questions: quantifying the welfare gains of switching to the new Ramsey policy, how taxes should respond to shocks in this setup, ... Also, so far we have worked with a preference specification capturing the aggregate margins of the willingness to substitute. With such a specification corporate shares are never traded in equilibrium and capital gains taxation plays no role. In order to understand better the properties of capital gains taxation we plan to extend our analysis to a heterogeneous agent environment.
References


Appendix

Proof of Proposition 1

We first prove (i). From the government’s first order conditions, we have
\[ \phi_i = \beta \phi_{i+1} \left[ 1 + f_{u, t+1} - \delta_u \right], \]
\[ \phi_i = \beta \phi_{i+1} \left[ 1 + f_{u, t+1} - \delta_u \right]. \]

From the firm’s and household’s first order conditions, we obtain
\[ u_{c, t} = \left( \frac{1 - \tau^d_{i+1}}{1 - \tau^d_{i}} \right) \beta u_{c, t+1} \left[ 1 + (1 - \tau^e_{i+1})(f_{m, t+1} - \delta_m) \right], \]
\[ u_{c, t} = \left( \frac{1 - \tau^d_{i+1}}{1 - \tau^d_{i}} \right) \beta u_{c, t+1} \left[ 1 + f_{u, t+1} - \delta_u \right]. \]

By comparing both sets of conditions, we find the following. First of all, the first two equations imply \( f_{m, t} - \delta_m = f_{u, t} - \delta_u \) for all \( t \geq 1 \). Second, note that a constant allocation does not necessarily imply constant taxes. Next, if the Ramsey policy converges to a steady state in finite periods, then the equality of marginal returns to both types of capital implies that the optimal corporate tax is zero at the steady state. See expression (11) in the main text. Moreover, working backwards, it is obvious that the corporate tax must be zero in all previous periods, included period 0.

We now prove (ii). For utility functions that are separable in consumption and leisure and homothetic in consumption, the government’s first order condition for consumption can be written as
\[ \phi_i = u_{c, t} \left[ (1 + \lambda) + \lambda \sigma \right], \]
where \( \sigma = \frac{u_{c_{t}, c_i}}{u_{c_{t}, c_i}} \) is the coefficient of relative risk aversion. Then, the first order conditions for tangible and intangible capital imply that \( 1 - \tau^d_{i+1} = 1 - \tau^d_{i} \), i.e. the optimal dividend tax is constant over time. Moreover, from inspection of (IC), it is clear that the initial dividend tax is lump-sum and should be set as high as possible. \( \blacksquare \)
Proof of Proposition 2

From equation (15), we see that a constant allocation implies that \( (1 - \tau^d_t)(1 - \tau^c_t) \) must be constant. Using this in equation (17), we find that dividend taxes are constant, which also implies constant corporate taxes. Next, as in the proof of Proposition 1, we compare the government’s first order conditions,

\[
\phi_t = \beta \phi_{t+1} \left( f_{m,t+1} + (1 - \delta_m) \left( I_{x,t+1}^m \right)^{-1} \right) I_{x,t}^m,
\]

\[
\phi_t = \beta \phi_{t+1} \left( f_{u,t+1} + (1 - \delta_u) \left( I_{x,t+1}^u \right)^{-1} \right) I_{x,t}^u,
\]

and the firm’s,

\[
u_{c,t} = \left( \frac{1 - \tau^d_{t+1}}{1 - \tau^d_t} \right) \beta u_{c,t+1} \left[ (1 - \tau^c_{t+1}) f_{m,t+1} + \tau^c_{t+1} \delta_m + (1 - \delta_m) \left( I_{x,t+1}^m \right)^{-1} \right] I_{x,t}^m,
\]

\[
u_{c,t} = \left( \frac{1 - \tau^d_{t+1}}{1 - \tau^d_t} \right) \beta u_{c,t+1} \left[ f_{u,t+1} + (1 - \delta_u) \left( I_{x,t+1}^u \right)^{-1} \right] I_{x,t}^u.
\]

The first two equations imply \( f_{m,t+1} + (1 - \delta_m) \left( I_{x,t+1}^m \right)^{-1} = f_{u,t+1} + (1 - \delta_u) \left( I_{x,t+1}^u \right)^{-1} \) for all periods \( t \geq 0 \). Then, if there exists a steady state allocation, then it is clear that the equality of marginal returns to both types of capital implies that the optimal corporate tax is zero at the steady state. Moreover, as before, working backwards, this implies that the corporate tax must be zero in all previous periods, included period 0.

To characterize dividend taxes, we see that for utility functions that are separable in consumption and leisure and homothetic in consumption, we again obtain

\[
\phi_t = u_{c,t} \left[ (1 + \lambda) + \lambda \sigma \right].
\]

Thus, as before, the optimal dividend tax is constant over time. Moreover, as we can see from equation (16) and our new implementability constraint (IC'), the initial dividend tax is now distortionary since it affects effort. Therefore, if the upper bound on the tax rate \( \tau^\text{max} \) is sufficiently high, then the optimal initial dividend tax must be lower than \( \tau^\text{max} \).

\[\blacksquare\]