



Procurement Design with Corruption

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Abstract

This paper investigates the design of optimal procurement mechanisms in the presence of corruption. After the sponsor and the contractor sign the contract, the latter may bribe the inspector to misrepresent quality. Thus, the mechanism affects whether bribery occurs. I show how to include bribery as an additional constraint in the optimal-control problem that the sponsor solves, and characterize the optimal contract. I discuss both the case of fixed bribes and bribes that depend on the size of the quality misrepresentation, and also uncertainty about the size of the bribe. In all cases, the optimal contract curtails quality not only for low efficiency contractors but also for the most efficient contractors. Implementation is also discussed.

Keywords: bribery; quality; contract design

JEL numbers: D82, D73, H57

1 Introduction

There has been growing interest in recent years in both the economic effects of corruption and also potential remedies for dealing with it. Corruption ranks high among the difficulties when organizing procurement in general and public procurement in particular.¹ Indeed, officials and contractors are often tempted by the gains from trading

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¹As an example, according to the Minister for State-Owned Enterprises, 80% of corruption and fraud in state-owned Indonesian companies occur in public procurement. (Transparency International, 2006.)

in illegal, or at least illegitimate, mutual favors. Bribes are often exchanged for manipulation of either the rules that select the price (and/or identity of the contractor) or the quality of the good delivered. In this paper, I study the design of optimal procurement mechanisms for the principal (sponsor) in the face of this latter form of manipulation.² I begin by showing that standard tools in mechanism design may be adapted to characterize and analyze this problem. As we discuss below, the effect of this sort of corruption is to transform an adverse selection problem into an adverse-selection/moral-hazard problem to which standard tools may be applied.

Consider the standard procurement design problem, where the principal cares about quality and price, and the cost of delivering quality is the contractor's private information. Quality cannot be directly observed, and the contract supervisor or inspector in charge of assessing delivered quality can be "bought". In the simplest bribe model, this can be done by paying a fixed bribe. The key, simple observation is that a (direct) mechanism may then be defined as a triple of functions that specifies not only the quality and the price for each 'type' of contractor, but also whether the contractor should bribe or not. This is the moral hazard dimension that bribery introduces. As with collusion-proofness (see Tirole, 1986), dealing with corruption then adds a new incentive compatibility constraint to the sponsor's choice. In this case, the constraint that guarantees that the contractor does not have incentives to disregard the instruction with respect to bribing.

I characterize optimal mechanisms in this scenario and show that bribing imposes a bound on the levels and range of quality that are implementable. In response, the principal gives up inducing any (non-trivial) quality for an interval at the low-efficiency tail of types. This is probably not surprising. What is more subtle is that bribery is also problematic at the other end of the type domain. Indeed, due to the possibility of bribing, the sponsor optimally curtails the quality that it contracts with the most efficient contractors. Obtaining higher quality from these types obviously requires

²Transparency International lists this as one of the four areas of increased risk in the procurement cycle, together with limited access to information, abuse of exceptions and the budget phase. See Transparency International (2006).

promising them a higher price. That, in turn, makes it more attractive for low-efficiency types to bribe the inspector in order to (claim high efficiency and so) contract high quality, while delivering less of it. Therefore, the higher the quality contracted with high-efficiency types the tighter the incentive constraint for lower-efficiency types.

Note that this incentive constraint is not local, but global. Indeed, bribing allows a contractor to target the highest price while avoiding the cost of delivering the corresponding highest quality. Thus, conditional on bribing, a contractor would (contract highest quality, i.e., would) claim to be the highest efficiency type.

When the cost of manipulation, i.e., the size of bribe, is fixed and common knowledge, the possibility of bribery binds, but bribery does not occur when the sponsor designs the optimal mechanism. The same would be true if there was uncertainty about the size of the required bribe, but the uncertainty was resolved only after the project had been completed. However, things are different if the uncertainty about the required bribe was resolved after contracting but before the project was completed. This is the case if, for instance, the identity (and inclination to gift exchanges) of the contract supervisor is revealed only after the contract is signed. Completely preventing bribery may not be in the sponsor's interest in that case. I characterize the optimal mechanism under this alternative model. In that mechanism, no type bribes with probability one, but bribery may occur with positive probability for all types. Indeed, after quality is contracted and the size of the required bribe learnt by the contractor, the contractor will always choose whatever is cheaper: bribing or delivering. If the necessary bribe happens to be very small, bribing will be cheaper than delivering any non trivial quality. Thus, bribery can be seen as a modification of the contractor's cost function. Other than that, the problem is technically very similar to the design problem in the absence of corruption. The effect of bribery is that, again, the sponsor optimally gives up inducing the low tail of efficiency types to deliver quality, and contracts lower quality with all types of contractor, even the most efficient one. In that sense, the effect of bribery is similar whether bribes are paid or not in equilibrium.

I also consider the case where the size of the bribe depends on the size of manipulation obtained from the inspector. If the required bribe is (sufficiently) concave in the

size of manipulation, the conclusions are similar to the known, fixed-bribe case. Indeed, a fixed bribe may be thought of as an extreme example of a concave bribe. Under sufficient concavity, the conclusions of the model are not affected: moral-hazard incentive compatibility is binding for the lowest efficiency types who may imitate the highest efficiency type (a global constraint). No bribes are paid in the optimal mechanism, and quality is curtailed at both ends of the type distribution.

The convex case, on the contrary, is similar to the fixed but uncertain bribe case. In particular, if the marginal cost of bribing is very low for very low levels of manipulation, all types will find it attractive to 'buy' some manipulation instead of delivering all the contracted quality. As a result, bribery for all types will also be part of the outcome of the optimal mechanism. Under some assumptions on the convexity of the bribe, this again is akin to a modification of the contractor's cost function.

The shape of the 'bribe', i.e., of the cost that bribing represents for the contractor may be determined by social traits. Indeed, different societies are characterized by different degrees of the incidence and tolerance of corruption.³ When it is very costly to lure an official into illegal transactions, bribes are naturally concave in the amount of manipulation. A moderate bribe may be ineffective to induce the official to cross the line, but once crossed it makes sense to make the effort worthwhile. Contrary, when graft is widely practiced and viewed as acceptable to some degree, bribes are better represented as convex in the size of manipulation. Small manipulation is probably safe, but even if the rule is to break the rule there are norms as to what does it mean to *really break* rules.⁴

After characterizing optimal mechanisms, I study implementation, and show that a simple menu of contracts, much in the spirit of Che (1993), does implement optimal procurement.

³Several theories have been proposed to explain differences in the prevalence of corruption in societies, both based on 'rational' behavior (e.g., based on reputation, cf. Tirole, 1996; or multiplicity of equilibria, c.f., Acemoglu, 1995) and on 'social norms'. In their study of the empirical relationship between inequality and corruption, You and Khagram (2005) argue that social norms and perceptions is a channel through which inequality affects corruption.

⁴Truex (2011) reports the results of a survey of Kathmandu residents that shows that respondents generally agree that large-scale bribery was unacceptable, but also that petty corruption, gift giving, and favoritism are much more acceptable.

Finally, I consider competition. That is, I allow for the presence of several, symmetric contractors capable of undertaking the sponsor's project. Typically, competition does not affect the choice of quality as a function of the contractor's type, although it does reduce the expected price for the sponsor conditioning on quality. This is still the case when corruption introduces only locally binding incentive constraints. Indeed, as we have noted, both in the uncertain and fixed bribe and in the convex bribe cases corruption is equivalent to a modification of the contractors' cost function. This is unaffected by the number of potential contractors, and so bribery does not affect this optimal relationship between quality and type. However, when the bribe is fixed and known or concave, and so optimal procurement calls for avoiding bribes, things are more subtle. Indeed, in these cases the fact that competition reduces the price that is necessary for obtaining high quality from high efficiency types implies that competition reduces the incentives for low efficiency types to imitate high efficiency ones. The result is that competition relaxes the global constraint imposed by corruption. Consequently, competition alleviates the quality distortion for high efficiency types. That is, competition not only reduces the price of quality for the sponsor but also allows the sponsor to obtain higher levels of quality from the same type of contractor.

One technical contribution of this paper is to solve a problem where incentive constraints bind not only locally. As we have noted before, in the fixed and concave bribe cases, incentive compatibility related to moral hazard (no bribing) means that no type should have incentives to imitate the most efficient. This constraint is most stringent for the least efficient type. The strategy that I follow for characterizing the optimal mechanism is to fix an arbitrary quality for the most efficient type. Given this arbitrary value, I then characterize the sponsor's problem as a free-time, fixed-endpoint control problem. The optimal mechanism is the solution to this problem when the quality selected for the most efficient type is also optimal. That is, when the sponsor's surplus in the solution to the optimal control problem is maximized with respect to that quality.

Corruption has received much attention in the economic literature at least since Rose-Ackerman (1975) pioneering paper. (See Aidt, 2003, for a survey.) On the effects

of corruption on procurement, auction theorists have sought to understand how manipulation and bribe-taking affect the allocation and surplus-sharing when particular mechanisms are used to allocate contracts or goods. The literature has studied specific ways (typically, bid readjustment) in which an agent acting on behalf of the principal may, in exchange for a bribe, favor a contractor in some particular auction mechanism. (See, among other, Lambert-Mogiliansky, and Verdier, 2005, Menezes and Monteiro, 2006, Burguet and Perry, 2007, Koc and Neilson, 2008, Arozamena and Weinschelbaum, 2009, or Lengwiler and Wolfstetter, 2010.) The literature has also considered the effects of manipulation when quality, and not only price, is an argument in the principal's objective function. (See Celentani and Ganuza, 2002, and Burguet and Che, 2004.) With the exception of Celentani and Ganuza (2002), in all these papers the agent's ability to manipulate is defined in reference to the particular mechanism itself. Thus, the analysis of remedies is limited in scope, and for good reason: the mere question of what is an optimal mechanism from the principal's point of view may be ill-defined in these settings. For example, assume that an agent in charge of running a sealed-bid, first-price procurement auction is able to learn the bids before they are publicly opened, and is also able to adjust downward one of the contractors' bid, if that bid is not already the lowest. This is the model studied in most of the papers previously mentioned. If instead, the procurement mechanism was an oral auction, this ability would be irrelevant. Even in a second-price auction, altering one contractor's bid would be of no value to that contractor. Perhaps in either of these other auction formats the agent may manipulate the outcome by other means. The question would then become what is the 'equivalent' to bid readjustment in those, or any, other auction formats? That is, the question of what constitutes an optimal procurement format, in the presence of corruption, takes us to a more complex problem: defining what a corrupt procurement agent may do under each of the possible mechanisms the principal may design.⁵

⁵Note that, if the corrupt agent's ability to bend the rules depends on the rules themselves, the problem for the principal is not one where standard tools in mechanism design may be used. Even the revelation principle is problematic, as equivalence of mechanisms cannot be predicated without reference to equivalence of manipulation. That is, given the lack of a simple way of establishing

To my knowledge, this is the first paper that investigates the *design* of procurement mechanisms in the presence of bribery when the mechanism affects the incidence of bribery. Note that, contrary to many of the papers cited in the previous paragraph, I consider corruption in the execution of the contract, not in the negotiation of its terms. This means that the design of the procurement mechanism may affect the incidence of corruption, but what manipulation means can be defined independent of the mechanism itself. In their insightful paper, Celentani and Ganuza (2002) do study the optimal mechanism with a corrupt agent who may (adjust bids and) misrepresent quality assessment. However, (potential) corrupt deals pre-date the principal's choice of mechanism, and so the mechanism cannot affect quality, if corrupt deals indeed took place. Moreover, the authors assume that information rents are determined as in the absence of bribery, justifying this assumption on the need for 'hiding' corrupt deals. Under these assumptions, the sponsor knows that she will pay for quality as if no corruption existed, but will obtain that quality with a fixed, exogenous probability lower than one. Therefore, the sponsor optimally induces lower quality from all types: the optimal mechanism coincides with the optimal mechanism in the absence of bribery when quality is multiplied by a constant equal to that exogenous probability of actually getting the contracted quality.

In the next section, I present the basic model with a fixed bribe and obtain the optimal mechanism for this case. Section 3 introduces uncertainty concerning the size of the bribe and characterizes the optimal mechanism for that case. In Section 4, I let the size of the bribe be a function of the amount of manipulation, and obtain conclusions both for the (sufficiently) concave and convex cases. Section 5 discusses implementation of the optimal procurement rules, and Section 6 discusses the effects of an increase in competition. Some concluding remarks close the paper. An appendix contains all proofs.

equivalence classes in the set of mechanisms, restricting attention to revelation games may simply be of no use.

2 The model

A sponsor procures a contract of fixed size and variable quality, $q \in [0, \infty)$. Quality q measures the sponsor's willingness to pay for the project. That is, the payoff for the sponsor when she pays P and receives a project with realized quality q is simply $q - P$. The sponsor's goal is to maximize the expected value of that payoff.

A contractor is able to undertake the project and deliver quality q at a cost $C(q; \theta)$, where $\theta \in [\underline{\theta}, \bar{\theta}]$ is the contractor's type and her private information. From the sponsor's point of view, θ is the realization of some random variable with cdf F and density f in $[\underline{\theta}, \bar{\theta}]$, and this is common knowledge. We postulate a differentiable C with $C_\theta, C_q > 0$, and $C_{q\theta}, C_{qq} > 0$. This guarantees that, in the absence of bribery, monotonicity of q with respect to θ is virtually all that is needed for implementation. Also, we assume that $C(0; \theta) = 0$, for all θ , and that

$$C_q(q; \theta) + C_{q\theta}(q; \theta) \frac{F(\theta)}{f(\theta)} \quad (1)$$

is increasing both in q and in θ . This guarantees interior (to monotonicity constraints) solution for the optimal mechanism in the absence of bribery. (See Che, 1993.)

Zero quality should be interpreted as a standard minimum quality, and a positive quality as quality obtained at an additional cost to the contractor. Likewise, a price of zero corresponds to a payment that equals the (common) cost of honoring the contract at a standard, minimum quality, $q = 0$.

Under observability of quality, and so without any role for the inspector, a direct mechanism is a pair (p, q) , where $p : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$ and $q : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+$. $p(\theta)$ represents the payment that the contractor gets and $q(\theta)$ represents the quality that the contractor provides. Both are functions of the contractor's type. The sponsor needs to consider only incentive compatible, individually rational, direct mechanisms. Among them, the optimal one is characterized by quality $q^{NB}(\theta)$ that satisfies

$$1 - C_q(q^{NB}(\theta); \theta) - C_{q\theta}(q^{NB}(\theta); \theta) \frac{F(\theta)}{f(\theta)} = 0. \quad (2)$$

as long as

$$q^{NB}(\theta) - C(q^{NB}(\theta); \theta) \geq C_\theta(q^{NB}(\theta); \theta) \frac{F(\theta)}{f(\theta)}, \quad (3)$$

and $q^{NB}(\theta) = 0$ otherwise. (See Che, 1993.) In order to simplify the analysis and also concentrate on the interesting results below, we will assume that (3) holds with equality when evaluated at $\bar{\theta}$ and $q^{NB}(\bar{\theta}) = 0$ so that $q^{NB}(\theta) > 0$, for all $\theta < \bar{\theta}$.⁶

In this paper, we are interested in analyzing the case when quality q is not directly observable. Instead, the sponsor requires an agent, the inspector, who assesses (or certifies) the quality of the completed project. The inspector can observe q without cost.

The inspector can also accept bribes to be untruthful when reporting on quality. For now, we assume that the agent will do so in exchange for a fixed bribe B . In later sections we will consider more general specifications of B .⁷ In any case, we should note that the bribe B includes any (expected) payment by the contractor associated with bribing the agent.

Under the threat of bribery, we may define a direct mechanism as a triple (p, q, b) , where p and q are defined and interpreted as above, and $b : [\underline{\theta}, \bar{\theta}] \rightarrow \{0, 1\}$. We interpret $b(\theta) = 1$ as an instruction to bribe and $b(\theta) = 0$ as an instruction not to bribe. Also, we should interpret $q(\theta)$ as the *contracted* quality, not necessarily the delivered one.⁸

Let

$$\Pi(\theta) = b(\theta) (p(\theta) - B) + (1 - b(\theta)) (p(\theta) - C(q(\theta); \theta))$$

The mechanism is implementable if it satisfies incentive compatibility (IC) and individual rationality (IR), as in any standard problem:

IR) For all $\theta \in [\underline{\theta}, \bar{\theta}]$, $\Pi(\theta) \geq 0$.

IC) For all $\theta, z \in [\underline{\theta}, \bar{\theta}]$, $\Pi(\theta) \geq p(z) - \min \{C(q(z); \theta), B\}$.

Incentive compatibility incorporates adverse selection and moral hazard constraints.

⁶If (3) is negative at $\bar{\theta}$ for $q = 0$, everything that follows applies with only redefining the domain of types to the interval where (3) is non negative. Also, if (3) is strictly positive at $\bar{\theta}$ and for $q = 0$, then the value of θ^c defined later may be equal to, not strictly smaller than, $\bar{\theta}$. That "corner" solution would not affect any other insight.

⁷In all cases, we do not consider the possibility of "extortion", that is, the possibility that the inspector asks for bribes in order to report the truth, threatening to otherwise report lower quality than what is actually delivered. Note that in this type of corrupt behavior the parties involved, inspector and contractor, have conflicting interest. This is contrary to the cases studied in this paper.

⁸For the moment, we do not need to specify delivered quality: if $b(\theta) = 1$, then delivered quality will be 0, and otherwise it will be $q(\theta)$.

In particular, for $b(\theta) = 0$, $\Pi(\theta) \geq p(\theta) - B$. That is, the contractor's profits should be at least as large as the profit she could get by simply bribing. Note that incentive compatibility implies that if $b(\theta) = 1$, then delivered quality will be 0, and so we do not need to specify delivered quality when $b(\theta) = 1$.

The problem for the sponsor is to design the direct, implementable mechanism (p, q, b) that maximizes her expected payoff, $E[q(\theta)(1 - b(\theta)) - p(\theta)]$.

When the bribe B is known and fixed, that optimal mechanism cannot include an instruction to bribe for any type of contractor. The contractor would still have incentives to report truthfully her type if, instead, the mechanism asked her to deliver 0 quality and offered her a price of $p(\theta) - B$. The sponsor can thus get the same quality for a lower price. Indeed,

Lemma 1 *For any IC, IR direct mechanism, (p, q, b) , there exists an IC, IR mechanism with $b(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$, so that $E[q(\theta)(1 - b(\theta)) - p(\theta)]$ is higher for the latter.*

Thus, in this basic setting, bribery does not occur in the optimal mechanism, so we can restrict attention to mechanisms with $b(\cdot) = 0$, but bribery does still impose restrictions on mechanisms for IC and IR to hold.

In particular, standard necessary conditions for IC and IR (under no bribery) are still necessary. Indeed, even without intending to bribe the inspector, the contractor should have incentives to report her type truthfully. Three consequences follow from these necessary conditions. First, $\Pi(\theta)$ must be monotone decreasing. In fact,

$$\pi(z; \theta) \equiv p(z) - C(q(z); \theta)$$

satisfies the conditions of Theorem 2 in Milgrom and Segal (2002), so that

$$\Pi(\theta) = p(\theta) - C(q(\theta); \theta) = \Pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(z); z) dz, \quad (4)$$

and is absolutely continuous. That virtually fixes $p(\theta)$ as a function of $q(\theta)$. Second, and as a result, a necessary but also sufficient condition for IR is $\Pi(\bar{\theta}) \geq 0$. Third, monotonicity can also be extended to q and p in a standard way.

Lemma 2 *In any IC mechanism, $p(\theta)$ and $q(\theta)$ are monotone decreasing.*

These features, which are shared by any feasible mechanism in the absence of corruption, have important consequences that make this problem tractable. Indeed, since p is monotone, a bribing contractor of any type would maximize profits by claiming type $\underline{\theta}$. Also, since Π is monotone, type $\bar{\theta}$ is the type with the strongest incentives to bribe. Thus, the only constraint imposed by bribery that binds is

$$\Pi(\bar{\theta}) \geq p(\underline{\theta}) - B.$$

That is, substituting for $p(\underline{\theta})$ from (4),

$$B \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q(z); z) dz. \quad (5)$$

The main insight in this section is contained in (5). Bribery constrains the variation allowed to $q(\theta)$. When corruption is absent, the sponsor optimally asks for higher quality when it is less costly to obtain (when θ is lower). However, increases in quality require faster increases in prices, and high differences in prices cannot be sustained when bribing is inexpensive. This puts a limit on the range of quality levels that can be commanded.

It is a quite standard result that these conditions are not only necessary but also sufficient for implementation, provided that $C_{q\theta}(q; \theta) > 0$. Thus, the problem that the sponsor solves is

$$\max_{p, q} \int_{\underline{\theta}}^{\bar{\theta}} \{q(\theta) - p(\theta)\} f(\theta) d\theta,$$

s.t. $\Pi(\bar{\theta}) \geq 0$; (4); (5) and subject to $q(\theta)$ being monotone decreasing.

IR binds in the solution to this problem.⁹ Thus, $p(\bar{\theta}) = C(q(\bar{\theta}), \bar{\theta})$. The constraint (5) may not bind. Indeed, if (5) holds for $q^{NB}(\theta)$, then this is also the optimal mechanism under bribery. However, the case of interest is when B is sufficiently small so that

⁹Indeed, for any mechanism (q, p) so that $\Pi(\bar{\theta}) = p(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) = \epsilon > 0$, we can define a mechanism (q', p') with $q' = q$ and $p' = p - \epsilon$ for all θ . The new mechanism satisfies all the above constraints and results in a higher payoff for the sponsor.

(5) does bind. The following proposition characterizes the optimal mechanism also for this case

Proposition 3 *If (5) holds for the optimal mechanism without bribery, $q^{NB}(\theta)$, then (q^{NB}, p^{NB}, b) with $b(\theta) = 0$ for all θ , is the optimal mechanism under bribery. Otherwise, there exist θ^a and θ^c , with $\underline{\theta} < \theta^a \leq \theta^c < \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^c$; (ii) $q(\theta) = q^{NB}(\theta)$ if $\theta \in (\theta^a, \theta^c)$; and (iii) $q(\theta) = q^{NB}(\theta^a)$ if $\theta < \theta^a$.*

Thus, when the size of the bribe is fixed and known and bribery is a relevant constraint, the sponsor gives up the possibility of obtaining (an increase in) quality from contractors with very high costs. This result is not surprising. Indeed, recall that bribery imposes a limit on the range of possible quality levels that can be commanded. If the sponsor is going to shave this range, she optimally does so for types with low efficiency. The loss from such distortion is lowest when the contractor has one of these types. But, perhaps less obviously, quality is optimally distorted also at the top, i.e., when the contractor has a high efficiency.

The intuition behind this result is simple. (See Figure 1.) Assume $B > C(q^{NB}(\underline{\theta}); \underline{\theta})$, but also assume that (5) binds for $q^{NB}(\theta)$.¹⁰ That is,

$$B < C(q^{NB}(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q^{NB}(z); z) dz.$$

It is then feasible to distort only at the bottom. That is, to select θ^c that solves $B = C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\theta^c} C_{\theta}(q(z); z) dz$, and let $q(\theta) = q^{NB}(\theta)$ for all $\theta \leq \theta^c$ and $q(\theta) = 0$ for all $\theta > \theta^c$. This is represented in Figure 1. The thick line represents $q^{NB}(\theta)$ and the thin line this $q(\theta)$. Now consider the effects of reducing $q(\theta)$ for all types in $[\underline{\theta}, \underline{\theta} + \varepsilon)$ to the level $q(\underline{\theta} + \varepsilon)$. Firstly, it implies obtaining a lower quality from types in $[\underline{\theta}, \underline{\theta} + \varepsilon)$ and consequently reducing the payment to those types of contractor, just as in the absence of bribery. This effect is of second order when $q(\theta) = q^{NB}(\theta)$. Secondly, it

¹⁰Of course, if $B < C(q^{NB}(\underline{\theta}); \underline{\theta})$ then distortion at the top is imposed by constraint (5) on any implementable mechanism.

is delivered.^{11, 12} In particular, assume now that B takes values in some interval, $[\underline{B}, \overline{B}]$ according to some c.d.f. G with density g . We will make the standard assumption that $\frac{1-G(B)}{g(B)}$ is decreasing in B .

To simplify the analysis, we may also assume that \underline{B} is sufficiently low, say $\underline{B} = 0$. (At the end of this section, we will comment on the consequence of not assuming so.) Under these conditions, an optimal mechanism is not bribe-proof in general. Indeed, preventing all types of contractor from bribing in all probability -when B is low- may be too expensive for the sponsor.

Thus, we should now define a contract as a triple (p, q, b) , where $p : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}$ and $q : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_+$, as before, and $b : [\underline{\theta}, \overline{\theta}] \rightarrow [\underline{B}, \overline{B}]$. $b(\theta)$ is now interpreted as the cut-off value for B so that type θ is instructed to bribe if and only if $B < b(\theta)$. It is sufficient to consider mechanisms of this form. Indeed, if a contractor prefers not to bribe when the size of the bribe is B , she will also prefer not to bribe when the size of the bribe is larger than B . Contrary, if the contractor prefers to bribe when the size of the bribe is B , then she also prefers to bribe if the size of the bribe is smaller than B . Thus, IC will require that the instruction to bribe takes a cut-off form. Also, as in the previous section, we do not have to specify what quality is to be delivered if bribing: IC requires that quality to be 0.

As we have mentioned, in general, bribing will take place in equilibrium, but there is still a counterpart to Lemma 1:

Lemma 4 *For any IC and IR mechanism where at least one type θ bribes with probability 1, there is another IC and IR mechanism where no type bribes with probability 1 and results in (weakly) higher payoff for the sponsor.*

Thus, we may restrict attention to mechanisms with $b(\theta) < \overline{B}$. Also, since $\underline{B} = 0$,

¹¹This would fit a case where the agent in charge of quality assessment is hired after the contract has been signed. Note that, if the contractor privately learnt B before signing the contract, we would have a standard mechanism-design problem where the contractor's "type" would be (θ, B) .

¹²The case where uncertainty about B is resolved after the quality is delivered is equivalent to the fixed and known B , with only substituting EB for B . Indeed, in that case the decision whether to bribe or not would have to be taken before providing quality. Not providing quality but claiming to do so would require the contractor to pay the bribe B whatever its realization, whereas if the contractor provides quality there is no point in also paying the bribe.

all types for which $q(\theta) > 0$ will bribe with positive probability.¹³ That is, $b(\theta) > \underline{B}$ for those types. Finally, we may restrict attention to mechanisms such that $\Pi(\bar{\theta}) = 0$. Given any other IC and IR mechanism, a reduction in p constant over θ would be feasible without a change in incentives and participation.

Assuming that the size of B is uncertain at the time of contracting changes the nature of the moral-hazard incentives for the contractor. Indeed, whatever quality the contractor has committed to, she will prefer to bribe if, and only if, the realized value of the bribe is less than the cost of providing that quality. Thus, if she reports her type truthfully, IC requires

$$b(\theta) = C(q(\theta), \theta). \quad (6)$$

This is a moral-hazard, incentive compatibility constraint and defines $b(\theta)$ once $q(\theta)$ is determined. Moreover, this constraint is equivalent to a modification of the contractor's (expected) cost function. Indeed, whatever quality q a contractor of type θ commits to, her expected cost of honoring this commitment is now

$$\Phi(q, \theta) = E_B \min \{C(q, \theta), B\}.$$

Therefore, (sufficient) uncertainty about the size of the bribe renders the problem of designing an optimal mechanism for procurement a standard one with a modified cost function. Consequently, we can use exactly the same tools that we use in the design problem without bribery. Let

$$\pi(z; \theta) = p(z) - \Phi(q(z), \theta).$$

This function again satisfies the conditions of Theorem 2 in Milgrom and Segal (2002), so that, since $\Pi(\theta) = \arg \max_z \pi(z; \theta)$, we can write

$$\Pi(\theta) = p(\theta) - \Phi(q(\theta), \theta) = \Pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(z); z) [1 - G(C(q(z), z))] dz. \quad (7)$$

As in the previous section, monotonicity of $q(\theta)$ and $\Pi(\theta)$ is necessary for implementation. Also, this together with (6), (7), and $\Pi(\bar{\theta}) \geq 0$ is sufficient as long as $\Phi_{q\theta}(q, \theta) > 0$.

¹³Note that, if $B = 0$, only zero quality could avoid bribery. Thus, unless the sponsor gives up any hope to obtain positive quality with positive probability, indeed all types will bribe with positive probability.

This condition requires $C_\theta(q; \theta) [1 - G(C(q; \theta))]$, and not only $C_\theta(q; \theta)$, to be increasing in q . It is satisfied, for instance, if B is disperse, so that g is small for any value of B .

The expected surplus for the sponsor can be written as

$$\int_{\underline{\theta}}^{\bar{\theta}} \{[1 - G(C(q(\theta), \theta))] [q(\theta) - C(q(\theta), \theta)] - \Pi(\theta)\} f(\theta) d\theta, \quad (8)$$

where, in an IC mechanism, $\Pi(\theta)$ is given in (7). We are now ready to discuss the optimal mechanism for the sponsor.

Proposition 5 *There exists θ^d , with $\underline{\theta} < \theta^d \leq \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^d$; and (ii) if $\theta < \theta^d$, then $q(\theta)$ solves*

$$0 = \frac{1 - G(C(q, \theta))}{g(C(q, \theta))} \left\{ 1 - C_q(q, \theta) - C_{\theta q}(q, \theta) \frac{F(\theta)}{f(\theta)} \right\} - C_q(q, \theta) \left\{ q - C(q, \theta) - C_\theta(q, \theta) \frac{F(\theta)}{f(\theta)} \right\}. \quad (9)$$

The intuition behind this result is relatively straightforward when considering the Hamiltonian of the problem that the sponsor's choice of q solves. Suppressing the variables in the functions for compactness, this Hamiltonian is

$$H = (1 - G(C)) [(q - C) f - FC_\theta].$$

Note that in the absence of bribery, the corresponding Hamiltonian would be

$$H^{NB} = (q - C) f - FC_\theta.$$

Thus,

$$H = (1 - G(C)) H^{NB}.$$

The interpretation is simple. Quality $q(\theta)$ will be delivered with probability $1 - G(C)$, and then with that probability, the decision on q will have the same consequences on contractor rents, cost, and sponsor surplus, as in the absence of bribery. This is the phenomenon captured in Celentani and Ganuza (2002). In the present setting, this would have no effect on q , but only on the prices, p .¹⁴ However, $q(\theta)$ will also affect

¹⁴In Celentani and Ganuza (2002), the effect on q appears from the fact that the price is determined by non-corrupt contractors, who are assumed to act as in the absence of corruption. Therefore, the only way of controlling bribery rents is by affecting quality.

the probability of bribery through its effect on the threshold $b(\theta) = C(q(\theta), \theta)$. This represents an additional effect of a small increase in $q(\theta)$ of

$$-g(C)C_q H^{NB} < 0.$$

Indeed, a (small) increase in q will increase by $g(C)C_q$ the probability that the sponsor gets 0 quality, instead of q , in exchange for the price. Thus, for values of q that solve $\frac{\partial H^{NB}}{\partial q} = 0$, the presence of this negative effect of a small increase in q would imply that $\frac{\partial H}{\partial q} < 0$.¹⁵ Thus, the solution under bribery implies lower values of q for any θ .

For high enough types, θ , so that that $q^{NB}(\theta)$ is positive but small, the optimal answer is to relinquish the possibility of obtaining any quality above the minimum. Just as in the fixed-bribe case, attempting to do so would be too expensive in terms of rents.

Also, as in the fixed bribe case, distortions at the top are part of the optimal mechanism. Indeed, for the most efficient type, $q(\underline{\theta})$ solves

$$\frac{1 - G(C)}{g(C)} [1 - C_q] = C_q [q - C].$$

The left hand side is zero at $q^{NB}(\underline{\theta})$, since under no bribery there is no distortion at the top. But the right hand side is positive at that value. Thus, indeed $q(\underline{\theta}) < q^{NB}(\underline{\theta})$.

In summary, an uncertain bribe is equivalent to a modification of the contractor's cost function. Local IC constraints are correspondingly modified, which implies quality distortions for all type of contractors, and bribe payment with positive probability.

We have assumed that the lower support of the distribution of B was sufficiently low so as to make it too expensive for the sponsor to avoid bribery completely, even for the most efficient type.¹⁶ If \underline{B} was larger, we could use the same techniques used in the proof of Proposition 6 to obtain the optimal mechanism, but this time the constraint would be $\Pi'(\theta) = -C_\theta(q(\theta), \theta) [1 - G(\min\{C(q(\theta), \theta), \underline{B}\})]$. Thus, in the solution to that problem, we will have $q(\theta) = q^{NB}(\theta)$ whenever $C(q^{NB}(\theta), \theta) < \underline{B}$. Note that

¹⁵Of course, if $G(C(q(\theta), \theta)) = 0$ when evaluated at the $q(\theta)$ that solves $\frac{\partial H^{NB}}{\partial q} = 0$, then bribery does not change $q(\theta)$.

¹⁶Also, if \underline{B} is sufficiently high, then bribery will not be binding.

as long as (2) defines a continuous and monotone $q^{NB}(\theta)$, that solution $q(\theta)$ is also continuous and monotone, so indeed it defines the optimal mechanism.

4 Bribe models

In the previous sections, we considered extremely simple models of bribery. The inspector was willing to alter her assessment of quality with no bounds, and in exchange for a fixed bribe. That model of bribery may appear too simplistic, but in this section I will show that the main findings extend to a considerably more general set of models.

Let us return to the deterministic case, where the cost of manipulation is common knowledge at the time of contracting. Suppose now that the cost of manipulation (bribe and any other cost) is increasing with the size of that misrepresentation. That is, a quality overstatement of size m requires a 'bribe' $B(m)$, with $B' > 0$.

Recall that B includes all costs born by the contractor as a consequence of quality misrepresentation. That is, this includes, among other things, possible liabilities in case of 'catastrophic failure' or bribe detection.

Before discussing this case, note that the fixed, known-bribe case in Section 2 is an extreme case of a concave, increasing $B(m)$. Indeed, there $B(0) = 0$, and $B(m) = B$ for all $m > 0$. Also, although more subtle, the fixed, unknown bribe case in Section 3 can be thought of as a convex, increasing $B(m)$. Indeed, a contractor (with type θ) who contracts delivery of quality q will buy expected manipulation $G(C(q; \theta))q$. The ratio of expected bribe to expected manipulation,

$$\frac{\int_0^{C(q; \theta)} Bg(B)dB}{G(C(q; \theta))q}$$

is increasing in q , and expected manipulation is also increasing in q .

This is indicative that the results in those sections may actually extend to the increasing, known case we treat here, depending on the curvature of $B(m)$. We show now that this is, in a precise sense, true.

But, what is the meaning of the curvature of B ? As argued in the Introductions, concave $B(m)$ occurs when, for instance, it is quite costly to engage the inspector in

illegal activities, but increasing degrees of manipulation can be obtained at decreasing additional cost.¹⁷ On the other hand, a convex $B(m)$ better represents realities where 'petty corruption' is a kind of accepted, or tolerated, social norm, but large misrepresentations may risk crossing the line. Thus, although corruption is a threat in all societies, the curvature of B may be linked to the extent to which honesty is a standard in the society.¹⁸

4.1 Concave $B(m)$

Let us first consider the concave case. Sufficient concavity, as obtained under the assumptions below, implies that, if bribing, the contractor will still do so to claim the most efficient type. That is, she will offer the highest quality by claiming the lowest type $\underline{\theta}$ and in fact deliver quality 0. Thus, once again corruption will introduce only one additional, global constraint similar to (5).

This is a set of sufficient assumptions for that to be the case:

$$A1) -B''(m) > C_{qq}(q - m; \theta) \text{ for all } m \leq q \text{ all } \theta, \text{ and all } q \leq q^{NB}(\underline{\theta}).$$

$$A2) B'(q^{NB}(\theta)) < C_q(q^{NB}(\theta); \theta) \text{ for all } \theta.$$

$$A3) B(q^{NB}(\theta)) > C(q^{NB}(\theta); \theta)$$

A3 simplifies the analysis: if it were violated for some type θ , then we would have to consider an additional constraint on the quality that could be obtained from θ . Under A1, for any type θ of the contractor and any (relevant) contracted quality level q , $C(q - m; \theta) + B(m)$ is concave and so will be minimized either without bribes, or with bribe $m = q$. A2 implies that $p^{NB}(\theta) - B(q^{NB}(\theta))$ is monotone in θ . We discuss later the consequences of violating A1 and A2.

Also, we will restrict attention to cases where $C(q^{NB}(\underline{\theta}); \bar{\theta}) > B(q^{NB}(\underline{\theta})) > C(q^{NB}(\underline{\theta}); \underline{\theta})$.

If the second inequality did not hold, then it would be impossible to obtain quality

¹⁷Transparency international (2006) notes: "Experience in industrial countries shows clearly that -apart from facilitation payments- the majority of corrupt people (both on the private and the government side) are not junior or subordinate staff, but people in the higher echelons, including many senior managers." This form of 'grand corruption' is probably best described by a concave bribe function.

¹⁸Obviously, the type of project may also affect the 'shape' -and size- of the bribe function. For example, the probability of catastrophic failure if quality is not as specified, may be concave -large risk for even small amounts of manipulation- or convex -lower but increasing sensitivity to manipulation.- in manipulation, and this may result in 'bribe' functions with corresponding shapes.

$q^{NB}(\underline{\theta})$, since even for type $\underline{\theta}$ it would be cheaper to bribe. Thus, the ceiling on quality would be exogenously imposed. If the first inequality did not hold, bribery would never be a concern. Therefore the only interesting cases are those that satisfy the two inequalities.

In this new setting, a direct mechanism must not only specify whether to bribe or not, but also how much to bribe. Since $B(\cdot)$ is invertible, this is equivalent to determining the degree of manipulation exerted by the inspector. Thus, we can represent a mechanism as a triple $(p, q, m) : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}^3$, where now $q(\theta)$ represents the *contracted* quality and $m(\theta)$ the amount of quality manipulation. That is, delivered quality will be $q(\theta) - m(\theta)$.

It is straightforward to extend Lemma 1 to this scenario.

Lemma 6 *For any IC, IR direct mechanism, (p, q, m) , there exists an IC, IR mechanism with $m(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$, so that $E[q(\theta) - m(\theta) - p(\theta)]$ is higher for the latter.*

Now, for a mechanism with $m(\theta) = 0$ for all θ , incentive compatibility requires that

$$\theta = \arg \max_z \{p(z) - C(q(z); \theta)\} \quad (10)$$

is satisfied. As we mentioned above, assumption A1 implies that if a type θ of contractor bribes to claim a type z , then optimally she will either buy manipulation $m = 0$ or $m = q(z)$. (10) implies that the first choice is never preferred to truthful reporting. Thus, we will only need to consider the case when the contractor fully bribes in order to claim a type other than θ .

Just as in Section 2, (10) also implies monotonicity of q , p , and $\Pi(\theta)$. Therefore, the type of contractor that has more incentives to bribe to claim any type θ so as to obtain profits $p(\theta) - B(q(\theta))$ is a contractor of type $\bar{\theta}$. In particular, IC imposes

$$\Pi(\bar{\theta}) \geq p(\underline{\theta}) - B(q(\underline{\theta})), \quad (11)$$

the equivalent of (5), which can also be written:

$$B(q(\underline{\theta})) \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q(z); z) dz.$$

Also, (4) still defines $p(\theta)$ once $q(\theta)$ is determined. Therefore, A2 and monotonicity of $q(\theta)$ guarantees that (11) is the only constraint that bribing imposes. Thus, to all effects, the bribe is still fixed and equal to $B(q(\underline{\theta}))$, *once $q(\underline{\theta})$ is determined*. Consequently,

Proposition 7 *Under concavity of $B(m)$, A1, A2, and A3, if $q^{NB}(\theta)$ violates (11) then there exist θ^a and θ^c , with $\underline{\theta} < \theta^a \leq \theta^c < \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^c$; (ii) $q(\theta) = q^{NB}(\theta)$ if $\theta \in (\theta^a, \theta^c)$; and (iii) $q(\theta) = q^{NB}(\theta^a)$ if $\theta < \theta^a$.*

The strategy of the proof of Proposition 3 was to discuss the properties of the solution to the sponsor's problem for an exogenously given $q(\underline{\theta})$. These properties carry over to the concave $B(m)$ case. The only difference with Section 2 is that now $q(\underline{\theta})$ affects also the left hand side of the no-bribe constraint (11). Yet, A2 still guarantees that $\frac{d\theta^c}{dq(\underline{\theta})} < 0$ when evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$, and so indeed $\theta^a > \underline{\theta}$.

We now discuss the role of our assumptions. First, note that concavity is all that is needed for Lemma 6 to hold. Thus, the optimal procurement mechanism induces no bribe in equilibrium as long as $B(m)$ is concave. Now, if A1 is not satisfied, then the best deviation for some type θ may involve a mixture of bribing and positive quality delivery. This, in turn, implies that (11) and (10) plus monotonicity do not guarantee IC for all types, and further upper constraints on q may apply. The consequence would be the need for 'ironing' on q .

Also, if A2 does not hold, then the best bribing deviation for any type of contractor may be to claim a type θ^m below the type θ^a obtained above. The optimal mechanism could still be constructed from $\bar{\theta}$ down at a cost of complexity. For instance, if $p^{NB}(\theta) - B(q^{NB}(\theta))$ is not monotone, but it is single peaked, the problem for the sponsor is very similar to the one considered in Proposition 3 in what refers to the interval of types $(\theta^m, \bar{\theta})$. There will be two types θ^a and θ^c in the interior of this interval and the optimal mechanism prescribes for types in this interval what Proposition 3 prescribes for the whole set of types. For types below θ^m , the local incentive compatibility constraint that defines $q^{NB}(\theta)$ and the global incentive-compatibility constraint associated to bribery

would both have to be satisfied. Therefore, we may construct the optimal $q(\theta)$ from θ^m down as follows: for any θ and given $q(\cdot)$ for $\theta' > \theta$ by setting $q(\theta)$, as the minimum of $q^{NB}(\theta)$ and the solution to

$$C(q; \theta) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(x); x) dx = B(q).$$

Summarizing, as in Section 2, concavity of B implies no bribe in the optimal mechanism, and under sufficient concavity (so that A1 and A2 are satisfied) bribery imposes a ceiling on the quality that the sponsor obtains from the contractor.

4.2 Convex $B(m)$

We now turn to the convex case and assume that $B(m)$ is convex with $B'(0) = 0$. Under this assumption, Lemma 6 no longer holds. Indeed, convexity of B together with convexity of C implies that $C(q - m, \theta) + B(m)$ is convex in m , for any q and any θ . That is, after *contracting* any level of quality, the contractor achieves cost minimization by partly bribing and partly delivering since $C_q(0; \theta) = B'(0) = 0$.

Given $q(\theta)$, incentive compatibility requires that $m(\theta)$ solves

$$\min_{m \in [0, q(\theta)]} C(q(\theta) - m; \theta) + B(m).$$

Thus

$$-C_q(q(\theta) - m; \theta) + B'(m) = 0, \tag{12}$$

implicitly defines optimal $m(\theta)$ as a function of quality $q(\theta)$. Equivalently, if $\hat{q} = q - m$ is actual delivered quality,

$$-C_q(\hat{q}; \theta) + B'(m) = 0. \tag{13}$$

defines m implicitly as an increasing function of delivered quality \hat{q} , given θ . Let this solution be $\hat{m}(\hat{q}; \theta)$. Note that $\hat{m}(\cdot)$ is increasing both in θ and \hat{q} . Then, as in Section 3, bribery may be thought of as a modification of the cost of quality for each type. This modified cost function is now

$$\hat{\Phi}(\hat{q}; \theta) = C(\hat{q}; \theta) + B(\hat{m}(\hat{q}; \theta)).$$

A delivered quality \widehat{q} will have a "cost" that is increased by the required bribe, $B(\widehat{m}(\widehat{q}; \theta))$ for a type θ . Once this cost modification is made, we face a standard problem with a standard solution. As such, monotonicity of $\widehat{q}(\theta)$ and $\Pi(\theta)$ is necessary for implementation. It is also sufficient, given the corresponding definition of $p(\theta)$ and (13) as long as $\widehat{\Phi}_{q\theta}(q, \theta) > 0$. A sufficient, but not necessary, condition for $\widehat{\Phi}_{q\theta}(q, \theta) > 0$ is that $\frac{B''}{B'}$ is decreasing. Thus, the following proposition is just a corollary of this discussion.

Proposition 8 *If $B(m)$ is convex, B'' decreasing, and $C_{qq\theta} > 0$, then there exists θ^d , with $\underline{\theta} < \theta^d \leq \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^d$; and (ii) if $\theta < \theta^d$, then $q(\theta)$ solves*

$$1 - C_q(\widehat{q}; \theta) - C_{\theta q}(\widehat{q}; \theta) \frac{F(\theta)}{f(\theta)} = \frac{\partial B(\widehat{m}(\widehat{q}; \theta))}{\partial \widehat{q}} + \frac{\partial^2 B(\widehat{m}(\widehat{q}; \theta))}{\partial \widehat{q} \partial \theta} \frac{F(\theta)}{f(\theta)}. \quad (14)$$

The conditions on B'' and $C_{qq\theta}$, guarantee that $\widehat{\Phi}_{\widehat{q}}(\widehat{q}; \theta) + \widehat{\Phi}_{\theta \widehat{q}}(\widehat{q}; \theta) \frac{F(\theta)}{f(\theta)}$ is increasing in \widehat{q} and θ , so that the solution to (14) is monotone.¹⁹

Note that the right hand side of (14) is positive, as we are assuming B'' to be decreasing. Thus, $\widehat{q}(\theta) < q^{NB}(\theta)$ when interior. Types above θ^d are optimally asked to deliver zero quality. Finally, for $\theta = \underline{\theta}$ the right hand side of (14) is still positive. That is, even for the most efficient type, realized quality is distorted downwards. Indeed, even for that type, realized quality is more expensive to 'produce' under bribery, since its cost is increased by the associated bribe. Thus, as in the case of random but fixed bribes, quality distortion is optimally introduced for all types.

5 Implementation

After having characterized the optimal procurement rules, we now consider 'implementation' of these rules with standard mechanisms. Consider the case of a fixed and known bribe, or the case where the required bribe is increasing and sufficiently concave in the amount of manipulation. We have shown that the optimal mechanism in

¹⁹If this is not increasing, then once again monotonicity would be violated by the solution to the first order conditions, and "ironing" would be needed.

this case coincides with the optimal mechanism under no corruption in the interior of some interval of types. For types above this interval, quality should be zero and for types below this interval, quality should be a constant. It is straightforward that such rules would be implemented by any mechanism that implements the optimal rules in the absence of corruption, with the only addition being a quality ceiling and a quality floor. In the jargon of Section 2, these would be $q^{NB}(\theta^a)$ and $q^{NB}(\theta^c)$, respectively. Thus, a menu of contracts similar to a first-score auction, will implement the optimal procurement: $(q, p(q))$ for $q \in (q^{NB}(\theta^c), q^{NB}(\theta^a))$, where θ^c and θ^a are defined in the corresponding previous sections, and for each of these quality levels, $p(q) = q - \Delta(q)$, where

$$\Delta(q) = q^{NB}(\theta^c) - C(q^{NB}(\theta^c), \theta^c) + \int_{q^{NB}(\theta^c)}^q C_{q\theta}(z; q_{NB}^{-1}(z)) \frac{F(q_{NB}^{-1}(z))}{f(q_{NB}^{-1}(z))} dz,$$

and where, q_{NB}^{-1} represents the inverse function of q^{NB} . Of course, the menu should be complemented with the contract $(p, q) = (0, 0)$. Following the analysis in the previous sections, this is a straightforward corollary of Proposition 4 in Che (1993).

Likewise, the optimal procurement rules when the size of the bribe is uncertain, and so optimally bribery takes place with some probability, may also be implemented with a menu of contracts. However, the distortion in assessment of quality $\Delta(q)$ is more complex in this case. Indeed, now,

$$\Delta(q) = q(\theta^c) - C(q(\theta^c), \theta^c) + \int_{q(\theta^c)}^q \Phi(z) dz,$$

where θ^c and $q(\theta)$ are defined in Proposition 5, and²⁰

$$\begin{aligned} \Phi(z) = & C_{q\theta}(z; q^{-1}(z)) \frac{F(q^{-1}(z))}{f(q^{-1}(z))} + \\ & C_q(z; q^{-1}(z)) \frac{g(C(z; q^{-1}(z)))}{1 - G(C(z; q^{-1}(z)))} \left\{ z - C(z; q^{-1}(z)) - C_\theta(z; q^{-1}(z)) \frac{F(q^{-1}(z))}{f(q^{-1}(z))} \right\}. \end{aligned}$$

(Note that $\Delta(q)$ affects the price, but not the tradeoff between bribing and delivering quality.) This is the menu of contracts that implements optimal procurement for the modified cost function Φ that we discussed in Section 3. Likewise, when B is a convex function of manipulation satisfying the properties of Section 4, optimal procurement may be achieved by offering the contractor the optimal menu of contracts under the modified cost function $\widehat{\Phi}$, discussed in that section.

6 Competition

Consider the fixed bribe case, but now assume that there are $n \geq 2$ symmetric potential contractors. A mechanism should now determine not only price, quality, and whether bribes should be paid, but also the identity of the contractor that is selected for the job. Still, under our assumptions, it is optimal to treat all contractors symmetrically and to avoid bribing. Moreover, under our assumptions, it is optimal to select the most efficient contractor. Let $x(\theta)$ represent the probability that a given contractor with efficiency parameter θ is selected and, for simplicity and without loss of generality, assume that payments are only made to the selected contractor. As in the case of one contractor, $\pi(z; \theta) \equiv x(z) [p(z) - C(q(z); \theta)]$ satisfies the conditions of Theorem 2 in

²⁰Similarly as in the proof of Proposition 4 in Che (1993), it is easy to check that the second order conditions are satisfied. Indeed, observe that $\Phi(z)$ evaluated at $q(\theta)$ is equal to the derivative of the sponsor's surplus with respect to q at the optimal solution in Proposition 6 minus

$$\frac{1 - G}{g}(1 - C_q).$$

The derivative of the sponsor's surplus with respect to q is zero for all θ , evaluated at the optimal solution. Thus, the partial of $\Phi(z)$ with respect to q^{-1} is positive at the optimal solution. Since q^{-1} is a decreasing function and the partial of $\Phi(z)$ with respect to z is negative, we conclude that $\Phi(z)$ is decreasing and so the second order conditions are satisfied.

Milgrom and Segal (2002), so that in an incentive compatible mechanism

$$\Pi(\theta) = x(\theta) [p(\theta) - C(q(\theta); \theta)] = \Pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} x(z) C_{\theta}(q(z); z) dz.$$

Then, (5) is replaced by²¹

$$B \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(z) C_{\theta}(q(z); z) dz,$$

where we have used $x(\underline{\theta}) = 1$. Bribery is not binding when this expression is satisfied for $q^{NB}(\theta)$. (Recall that $q^{NB}(\theta)$, the optimal quality in the absence of bribery, is independent on the number of contractors.) Thus, the presence of $x(z) = [1 - F(z)]^{n-1}$ in the expression above implies that bribery is a less serious constraint when $n > 1$. Moreover, as $x(z)$ is decreasing in n , the stronger the competition, i.e., the larger n , the larger the chances that bribery is not an issue. That is, competition may eliminate the threat of bribery.

The intuition is simple. In the absence of corruption, (profits and) prices are lower the stronger the competition, and so are the incentives to bribe in order to fetch those prices, when bribing is a possibility.

Perhaps more interesting, even when bribery binds, so that the expression above is violated, the constraint imposed by bribery is weaker the larger the value of n . Indeed, as when $n = 1$, $q(\theta)$ is distorted down to $q^{NB}(\theta^a)$ in an interval $[\underline{\theta}, \theta^a]$. The optimal value of θ^a , i.e., of $q(\underline{\theta})$, balances the loss of efficiency in that interval with the gain from obtaining second-best quality from types around θ^c . For $n = 1$, these effects are captured by the second and the first lines, respectively, in (21) in the proof of Proposition 3. When $n > 1$, these effects are still represented by (21) except that the first line is multiplied by $x(\theta^c) = [1 - F(\theta^c)]^{n-1}$ (and the second line is multiplied by $x(\underline{\theta}) = 1$). Thus, the larger n the lower the optimal value of θ^a , i.e. the smaller the range of types at the top for which quality is distorted.

²¹Incentive compatibility requires that $\pi(z; \theta)$ is maximized at $z = \theta$, which implies that, if x is monotone decreasing and q is constant, still $p(\theta)x(\theta)$ is decreasing in θ . Note that, in case of bribing, a contractor obtains an expected profit of $p(\theta)x(\theta) - B$, and so again when bribing the contractor will claim type $\underline{\theta}$.

Once again the intuition is simple. As we have already mentioned, the only reason to distort quality at the top is to reduce the incentives for lower efficiency types to bribe so that positive quality may be obtained from these types. When there are n contractors, this gain is realized from a contractor only with probability $[1 - F(\theta^c)]^{n-1}$, that is, the probability that there is no other contractor with a more efficient type. This probability is obviously smaller the stronger the competition, and so is the return of distortions at the top.²²

Consider now the case when B is uncertain at the time of contracting. Even if $n > 1$, (6) is an incentive compatibility condition. Thus, as with $n = 1$, bribery is formally equivalent to a modification of the cost function for contractors, where the new cost function is again $\Phi(q, \theta)$ defined in Section 3. This modified function is independent of n . Thus, the quality in the optimal mechanism is also independent of n , and so competition leaves that unaffected.

Indeed, as we discussed in Section 3, in this case the binding incentive compatibility constraints are all local. The tradeoff between bribing and delivering quality is unaffected by competition, although, as in the absence of bribery, competition reduces the price that the sponsors pays for that quality.

Thus, following our discussion in Section 4, competition relaxes the global constraint imposed by bribery, so that when it is optimal to avoid corruption this can be done at a lower efficiency cost. However, when bribery is simply part of the equilibrium outcome, competition does not affect this equilibrium phenomenon. Competition increases the sponsor's expected surplus, but only in as much as it increases the chances of selecting a more efficient contractor.

7 Concluding remarks

We have characterized the optimal procurement rules when the contractor may bribe the inspector so that the latter manipulates quality assessment once the contract be-

²²The direct effect of n on θ^c is also positive. However, the increase in n induces an increase in $q(\underline{\theta})$ which has an additional, negative effect on θ^c . However, in expected terms, the overall effect on quality is positive.

tween contractor and sponsor has been signed. Even when it is optimal to prevent bribery, the optimal rules distort quality downwards, not only for low-efficiency types, but also for the most efficient contractors. This is the case when the bribe is of known and of fixed size or, more generally, under sufficient concavity of the bribe necessary to secure a given level of manipulation. In these cases, bribery imposes a global (as opposed to local) incentive-compatibility constraint, which compresses the variability of quality that may be obtained. Thus, the sponsor optimally sets quality floors, but also quality ceilings, to what absent corruption would be optimal.

When the bribe is sufficiently convex, or it is uncertain at the time of contracting, totally avoiding bribery may be too costly, and so the optimal mechanism is characterized by some manipulation and bribe payments in equilibrium. In these cases, bribery simply affects local incentive-compatibility constraints. Still, quality is curtailed as a means of reducing the incidence of manipulation.

We have also considered menu of contracts that implement these rules. When bribery sets a global constraint, these are straightforward modifications of an optimal menu of contracts in the absence of bribery. The design of the menu is a little more involved when bribery sets local incentive constraints. Finally, we have discussed the effects of contractor competition. On the one hand, when bribery imposes a global constraint, this constraint is weakened as the number of potential contractors increases. Correspondingly, quality distortions are reduced. On the other hand, when only local constraints matter, so that bribery is part of the optimal outcome, the number of contractors does not affect the quality contracted with each type of contractor, and so competition does not help reduce quality distortions.

Throughout the paper, we have taken a parsimonious, reduced-form approach to bribery. All that we have modelled of the corrupt dealings between the contractor and the inspector was the contractor's payment/cost for such dealings, perhaps as a function of manipulation. This reduced form is appropriate when the contractor has all the bargaining power when dealing with the inspector. Future research should explore two related phenomena that are absent in the present paper.

First, the size of the bribe may be the result of more elaborate bargaining methods

between the inspector and the contractor. In that case, the sponsor's decision on $q(\theta)$ may affect the outcome of these negotiations. Under some conditions, the results in this paper may be extended quite straightforwardly. For instance, we could extend the results in the fixed, known bribe case of Section 2 assuming Nash-bargaining between the inspector and the contractor. Assume that the sum of the inspector's and contractor's expected costs of illegal dealings and misreporting is $2B$. As in Section 2, the optimal mechanism should prevent bribery. Then, as in Section 2, taking into account IC for non-bribing behavior, if a type θ contractor plans to claim type $\theta' < \theta$ and bribe, then her payoffs will be

$$C(q(\theta'); \theta') + \int_{\theta'}^{\bar{\theta}} C_{\theta}(q(z); z) dz - \left(\frac{C(q(\theta'); \theta) + 2B}{2} \right).$$

The second term is the bribe payment/cost for the contractor. Under some conditions, e.g., if

$$2C_{\theta}(q^{NB}(\theta); \theta) - C_{\theta}(q^{NB}(\underline{\theta}); \theta) > 0$$

for all θ , then bribery still imposes only a global constraint: the lowest efficiency type who delivers a positive quality, θ^c , should not have incentives to claim the highest efficiency type $\underline{\theta}$. In other terms, (5) should be replaced with

$$B \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\theta^c} C_{\theta}(q(z); z) dz - \frac{C(q(\underline{\theta}); \theta^c)}{2}.$$

Other than that, the analysis in Section 2 still applies. However, in more complex bargaining and/or bribing models, the interaction between local and global constraints may be also more complex.

Second, once we explicitly model negotiations between contractor and inspector, information issues may be important. The decision on $q(\theta)$ may affect not only the split of surplus between inspector and contractor, but also whether their illegal transaction takes place. Indeed, q will become a signal of θ in a negotiation over the bribe between contractor and inspector. The informativeness of this signal depends, at least in part, on the mechanism that the sponsor designs. Here, as in Baliga and Sjöström (1998), the

sponsor (principal) benefits from designing a mechanism that preserves the asymmetry of information between inspector and contractor about the type of the latter. Moreover, as in Celik (2009), the mechanism also determines the contractor's disagreement utility. Both things, information asymmetry and disagreement utility, affect the gains from trading bribes for favorable reports.

8 References

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9 Appendix

9.1 Proof of Lemma 1

Given a IC, IR mechanism (p, q, b) (p', q', b') where $b'(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$; $q'(\theta) = q(\theta)$, $p'(\theta) = p(\theta)$ if $b(\theta) = 0$; and $q'(\theta) = 0$, $p'(\theta) = p(\theta) - B$ if $b(\theta) = 1$. It is straightforward that $\Pi(\theta)$ is the same under both mechanisms for all θ . Also, $p'(z) - \min_z \{C'(q(z); \theta), B\} = p(z) - \min_z \{C(q(z); \theta), B\}$ if $b(\theta) = 0$, and $p'(z) - \min_z \{C'(q(z); \theta), B\} = p(z) - B \leq p(z) - \min_z \{C(q(z); \theta), B\}$ if $b(\theta) = 1$, and so the mechanism is incentive-compatible and individually rational. Finally, the sponsor's payoff is larger under (p', q', b') if the probability that $b(\theta) = 1$ is positive.

9.2 Proof of Lemma 2

IC applied to types $\theta + \Delta$ and θ , require $\pi(\theta; \theta) \geq \pi(\theta + \Delta; \theta)$ and $\pi(\theta; \theta + \Delta) \leq \pi(\theta + \Delta; \theta + \Delta)$, which simplifies to

$$0 \leq C(q(\theta + \Delta), \theta) - C(q(\theta + \Delta), \theta + \Delta) - (C(q(\theta), \theta) - C(q(\theta), \theta + \Delta)),$$

so that and since $C_{q\theta} > 0$, we conclude that $q(\theta)$ is indeed monotone decreasing. We may bound $q(\theta)$ without loss of generality, and thus, both $\Pi(\theta)$ and $q(\theta)$ are a.e. differentiable, and from (4) so is p . Also using (4), at any differentiability point of $\Pi(\theta)$,

$$p'(\theta) = C_q(q(\theta), \theta)q'(\theta) \leq 0. \tag{15}$$

At any non-differentiability point, we can rule out a jump upwards from continuity of $\Pi(\theta)$ and $C(q; \theta)$, and monotonicity of $q(\theta)$.

9.3 Proof of Proposition 3

The proof when (5) holds for the optimal mechanism without bribery is trivial. Now, assume that (5) is violated by the optimal mechanism without bribery, and consider

the following free-time, fixed-endpoint control problem

$$\begin{aligned} & \max_{q(\theta) \in [0, q(\underline{\theta})]} \int_{\underline{\theta}}^{\tau} \{q(\theta) - C(q(\theta); \theta) - X(\theta)\} f(\theta) d\theta \quad (16) \\ \text{s.t. } & X' = -C_{\theta}(q(\theta); \theta), \end{aligned}$$

with initial condition $X(\underline{\theta}) = B - C(q(\underline{\theta}); \underline{\theta})$ and target point $X(\tau) = 0$, for some given parameter $q(\underline{\theta})$. Note that

$$X(\theta) = X(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} X'(z) dz = \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q(z); z) dz \quad (= \Pi(\theta)). \quad (17)$$

The quality choice q^* in the optimal mechanism ($p^*, q^*, b^* = 0$) with $p^*(\theta)$ defined by (4) is a solution to this optimal control problem for $q(\underline{\theta}) = q^*(\underline{\theta})$ if it is monotone. Also, at that solution $q(\theta) = 0$ for $\theta > \tau$. The Hamiltonian for this optimal control problem is

$$H(\mu, X, q) = \{q(\theta) - C(q(\theta); \theta) - X(\theta)\} f(\theta) + \mu(-C_{\theta}(q(\theta); \theta))$$

where μ is the costate variable. Necessary conditions for interior solution include:

$$\begin{aligned} \mu' &= f(\theta), \\ -\mu C_{\theta q}(q(\theta); \theta) + (1 - C_q(q(\theta); \theta)) f(\theta) &= 0. \end{aligned}$$

Integrating for μ' , we obtain $\mu = F(\theta)$, and substituting in the second equation, we obtain the same marginal condition (2) as without bribery for values of $\theta < \tau$ at points where the solution in q is interior to $[0, q(\underline{\theta})]$. Given our assumptions, this solution to (2) is monotone decreasing. Therefore, if $q^{NB}(\theta) > q(\underline{\theta})$ at some θ , then the solution to the control problem at that θ is at a corner, $q(\underline{\theta})$, and when $q^{NB}(\theta) < q(\underline{\theta})$ but $\theta < \tau$, then q^* coincides with q^{NB} . Thus, defining θ^a as

$$q^{NB}(\theta^a) = q(\underline{\theta}), \quad (18)$$

the solution for $\theta < \theta^a$ is $q(\theta) = q(\underline{\theta})$. If $X(\tau) = 0$ binds ($\tau < \bar{\theta}$ at the solution of the control problem), and from (17) then $\tau \equiv \theta^c$, satisfies

$$B - C(q^*(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_{\theta}(q^*(\underline{\theta}); z) dz - \int_{\theta^a}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz = 0. \quad (19)$$

We now characterize the value $q^*(\underline{\theta})$ in the optimal mechanism, and so the values of θ^a and θ^c at the optimal mechanism. The sponsor's expected surplus in the solution to the control problem for a given value of $q(\underline{\theta}) \leq q^{NB}(\underline{\theta})$ is:

$$\int_{\underline{\theta}}^{\theta^a} \left\{ q(\underline{\theta}) - C(q(\underline{\theta}); \theta) - \int_{\theta}^{\theta^a} C_{\theta}(q(\underline{\theta}); z) dz - \int_{\theta^a}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz \right\} f(\theta) d\theta + \int_{\theta^a}^{\theta^c} \left\{ q^{NB}(\theta) - C(q^{NB}(\theta); \theta) - \int_{\theta}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz \right\} f(\theta) d\theta, \quad (20)$$

where θ^a is a function of $q(\underline{\theta})$ defined in (18), and θ^c is then also a function of $q(\underline{\theta})$ defined in (19). The derivative of (20) with respect to $q(\underline{\theta})$ is

$$\frac{d\theta^c}{dq(\underline{\theta})} \left\{ [q^{NB}(\theta^c) - C(q^{NB}(\theta^c); \theta^c)] f(\theta^c) - C_{\theta}(q^{NB}(\theta^c); \theta^c) F(\theta^c) \right\} + \int_{\underline{\theta}}^{\theta^a} \left\{ 1 - C_q(q(\underline{\theta}); \theta) - \int_{\theta}^{\theta^a} C_{\theta q}(q(\underline{\theta}); z) dz \right\} f(\theta) d\theta. \quad (21)$$

When evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$, and so at $\theta^a = \underline{\theta}$, the second line vanishes. Also, from (19), $\frac{d\theta^c}{dq(\underline{\theta})} < 0$. The curly brackets in the first line is positive if $q^{NB}(\theta^c) > 0$. Note that indeed $q^{NB}(\theta^c) > 0$ for θ^c evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$ since otherwise (5) would hold. Therefore, $q^*(\underline{\theta}) < q^{NB}(\underline{\theta})$. Also, changing the order of integration, the second line in (21) is

$$\int_{\underline{\theta}}^{\theta^a} \{ [1 - C_q(q(\underline{\theta}); \theta)] f(\theta) - C_{\theta q}(q(\underline{\theta}); \theta) F(\theta) \} d\theta > 0,$$

where the inequality follows from the fact that $q(\underline{\theta}) < q^{NB}(\theta)$ for all $\theta \in [\underline{\theta}, \theta^a]$ and the integrand is zero evaluated at $q = q^{NB}(\theta)$. Thus, at the optimal mechanism

$$[q^{NB}(\theta^c) - C(q^{NB}(\theta^c); \theta^c)] f(\theta^c) - C_{\theta}(q^{NB}(\theta^c); \theta^c) F(\theta^c) > 0,$$

and so $\theta^c < \bar{\theta}$, since we are assuming that $q^{NB}(\bar{\theta}) = 0$.

9.4 Proof of Lemma 4

As before, assume that a type θ^b bribes with probability 1. Then her profits are $p(\underline{\theta}) - EB$. That implies that any type $\theta > \theta^b$ also bribes with probability 1, or else

$q(\theta) = 0$. Indeed, $\Pi(\theta) = \Pi(\theta^b)$, since type θ can always imitate type θ^b and obtain the same profits, $p(\underline{\theta}) - EB$. But if $q(\theta) > 0$ and $b(\theta) > \underline{B}$, then θ^b would benefit from deviating and imitating type θ , since her costs are lower. Thus, assume that for all types $\theta \geq \theta^b$ either $q(\theta) = 0$ or $b(\theta) = \underline{B}$. Define (p', q', b') where $(p'(\theta), q'(\theta), b'(\theta)) = (p(\theta), q(\theta), b(\theta))$ for all $\theta < \theta^b$, but $(p'(\theta), q'(\theta), b'(\theta)) = (p(\underline{\theta}) - EB, 0, \overline{B})$ for $\theta > \theta^b$. Note that, conditional on truth-telling, the profits of all types are unchanged. Moreover, the profits of any deviating type are still the same, and so (p', q', b') is IC and IR. On the other hand the payoff for the sponsor is larger, weakly so if the the measure of types $\theta > \theta^b$ with $b(\theta) = \underline{B}$ is zero.

9.5 Proof of Proposition 5

Consider the following optimal control problem

$$\begin{aligned} & \max_{q(\theta) \geq 0} \int_{\underline{\theta}}^{\overline{\theta}} \{[1 - G(C(q(\theta), \theta))] [q(\theta) - C(q(\theta), \theta)] - X(\theta)\} f(\theta) d\theta \\ \text{s.t. } & X' = -C_\theta(q(\theta), \theta) [1 - G(C(q, \theta))], \end{aligned}$$

with initial condition $X(\overline{\theta}) = 0$. The quality q^* that maximizes (8) subject to (7) and (6) is also a solution to this control problem if it satisfies IC. The Hamiltonian of the problem is

$$\begin{aligned} H &= \{[1 - G(C(q, \theta))] [q - C(q, \theta)] - X(\theta)\} f(\theta) \\ &+ \mu [-C_\theta(q, \theta) [1 - G(C(q, \theta))]], \end{aligned}$$

so that again, for interior solution,

$$\mu' = f(\theta),$$

and so $\mu = F(\theta)$ and $\frac{\partial H}{\partial q} = 0$ can be written as

$$\begin{aligned} 0 &= \frac{1 - G(C(q, \theta))}{g(C(q, \theta))} \left\{ 1 - C_q(q, \theta) - C_{\theta q}(q, \theta) \frac{F(\theta)}{f(\theta)} \right\} \\ &- C_q(q, \theta) \left\{ q - C(q, \theta) - C_\theta(q, \theta) \frac{F(\theta)}{f(\theta)} \right\}. \end{aligned} \tag{22}$$

The right hand side is decreasing in q and θ , under our assumption that (1) is increasing both in q and in θ , and $\frac{1-G(\cdot)}{g(\cdot)}$ is decreasing. Thus, the solution $q(\theta)$ is also decreasing.

Assume that there exists θ' that solves

$$0 = 1 - C_q(0, \theta) - C_{\theta q}(0, \theta) \frac{F(\theta)}{f(\theta)} + g(0)C_\theta(0, \theta)C_q(0, \theta) \frac{F(\theta)}{f(\theta)}.$$

Then, $\theta^d = \theta'$, and the solution is $q(\theta) = 0$ for all $\theta > \theta^d$. Otherwise, $\theta^d = \bar{\theta}$. For $\theta < \theta^d$, (22) has a solution $q(\theta) > 0$.

9.6 Proof of Lemma 6

Assume $m(\theta) > 0$ for some value θ , and consider a change in the mechanism so that $q'(\theta) = q(\theta) - m(\theta)$, $m'(\theta) = 0$, and $p'(\theta) = p(\theta) - B(m(\theta))$. The profits of type θ do not change. Also, a type θ' imitating type θ could achieve

$$p(\theta) - \min_{z \in [0, q(\theta)]} \{C(q(\theta) - z; \theta') + B(z)\},$$

with the original mechanism, whereas with the modified mechanism she can obtain

$$\begin{aligned} & p'(\theta) - \min_{z \in [0, q'(\theta)]} \{C(q'(\theta) - z; \theta') + B(z)\} \\ &= p(\theta) - B(m(\theta)) - \min_{z \in [0, q(\theta) - m(\theta)]} \{C(q(\theta) - m(\theta) - z; \theta') + B(z)\} \\ &= p(\theta) - \min_{z \in [0, q(\theta) - m(\theta)]} C(q(\theta) - m(\theta) - z; \theta') + B(z) + B(m(\theta)) \\ &= p(\theta) - \min_{h \in [m(\theta), q(\theta)]} C(q(\theta) - h; \theta') + B(h - m(\theta)) + B(m(\theta)). \end{aligned}$$

where we have used the change of variable $h = z + m(\theta)$. This expression is smaller since B is concave and the choice set of h is smaller than the choice set of z in the original mechanism. The profits of θ' imitating any other type have not changed, and the profits of θ imitating any other type are not larger.

9.7 Proof of Proposition 7

The proof parallels that of Proposition 3, given $q(\underline{\theta})$. Thus, we need only show that the sponsor's surplus is maximized for $q(\underline{\theta}) < q^{NB}(\underline{\theta})$. The sponsor's objective is still given by (20), and so its derivative at $q^{NB}(\underline{\theta})$ is also given by (21). Then, we only need

show that $\frac{d\theta^c}{dq(\underline{\theta})} < 0$. Totally differentiatin the equivalent now to (19),

$$B(q(\underline{\theta})) - C(q(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_{\theta}(q(\underline{\theta}); z) dz - \int_{\theta^a}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz = 0,$$

we have

$$\frac{d\theta^c}{dq(\underline{\theta})} = \frac{B'(q(\underline{\theta})) - C_q(q(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_{\theta q}(q(\underline{\theta}); z) dz}{C_{\theta}(q^{NB}(\theta^c); \theta^c)} < 0,$$

where the inequality follows from A2 and the fact that $C_{\theta q}(q; \theta) > 0$.