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**Ignorance Promotes Competition: An Auction Model
with Endogenous Private Valuations**

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IGNORANCE PROMOTES COMPETITION:
AN AUCTION MODEL WITH ENDOGENOUS PRIVATE VALUATIONS *

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ABSTRACT

We study a situation in which an auctioneer wishes to sell an object to one of N risk-neutral bidders with heterogeneous preferences. The auctioneer does not know bidders' preferences but has private information about the characteristics of the object, and must decide how much information to reveal prior to the auction. We show that the auctioneer has incentives to release less information than would be efficient and that the amount of information released increases with the level of competition (as measured by the number of bidders). Furthermore, in a perfectly competitive market the auctioneer would provide the efficient level of information.

KEYWORDS: Auctions, Private Values, Asymmetric information.

JEL classification numbers: D44, D82, D83.

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“Lady Brandon treats her guests exactly as an auctioneer treats his objects. She either explains them entirely away, or tells one everything about them except what one wants to know” Oscar Wilde (1891) *The Picture of Dorian Gray*, reprint in Penguin Books, London, 1994, p. 14.

1 INTRODUCTION

One of the best known models in auction theory is the private value model. In this model, an auctioneer wants to sell an object to one of N bidders with possibly different valuations. Bidders know their valuation of the object but do not know the value of the object to other bidders. Typically, bidders' valuations are made up of two elements: bidders' preferences over characteristics, which are exogenous and privately known, and bidders' prior information on object characteristics. Most of the papers that analyze this model take bidders' information about the object as given and therefore assume that bidders' valuations are independently drawn from an exogenous distribution. In practice, there are many situations in which the auctioneer might control the amount of information about the object that is held by the market. In this case, the auctioneer does not regard the distribution of valuations as exogenous and must decide how much information about object characteristics he provides to the market taking into account the strategic effect that this information has on bidders' valuations. In this paper we study a situation in which bidders' tastes over characteristics are private information to the bidders but the object characteristics are private information of the auctioneer. We first show how bidders' valuations are endogenously determined by the information available on the characteristics of the object. Then, we focus on the auctioneer's decision about how much information he reveals to the bidders and solve for the efficient and optimal level of information provided.

The way we model this is as follows: consider an auctioneer who wishes to sell a single object to one of N risk-neutral bidders using a second-price sealed-bid auction. The auctioneer wants to maximize revenue and has private knowledge about the characteristics of the object to be sold. The object characteristics can be summarized as a point in an abstract product space. The bidders are horizontally differentiated. Each bidder has an ideal point in this product space and

his object valuation is decreasing in the distance between the location of his ideal point and the location of the object. Bidders' preferences about the characteristics of the object (the location of their ideal points) are private information. Prior to the bidding process the auctioneer decides the amount of information he releases to the market, anticipating how this information affects bidders' valuations of the object. The information provided by the auctioneer relates to the location of the object in the product space. When the auctioneer provides more information, bidders estimate the location of the object in the product space more precisely. Once the information has been disclosed and the bidders learn their expected valuation of the object, the model proceeds as a standard private valuations auction. The reason for this is that: (i) the auctioneer can control the accuracy of the bidders' inference process but he cannot observe bidders' valuations (since he does not know their preferences); (ii) bidders know their own expected valuations but do not know the expected value of the object to other bidders, since they do not observe their preferences.

This setup applies to situations in which there is asymmetric information on both sides of the market with respect to the value of the object being traded: the seller knows more about the characteristics of the object being sold and buyers have private information about their preferences. Two such situations spring to mind: the sale of a company, and certain kinds of internet auctions. The owner of a company, for example, is likely to have more information about the characteristics of the company than potential buyers, but idiosyncratic factors of potential buyers (such as distribution channels, corporate culture, technological or productive complementarities, ...) determine the synergies that arise from the purchase and hence the value of the company to the buyer.¹ As a result, the owner of the company must decide how much information to reveal to the bidders without having directly observed these idiosyncratic factors nor, consequently, bidders' preferences over characteristics. Bidders, for their part, are likely to react asymmetrically to the information provided by the auctioneer. For example, if the owner reveals information about

¹When analyzing company takeovers, it seems natural to incorporate a common value component as well as a private value one. Although in this paper we focus exclusively on private valuation auctions, ignoring the common value component, our model can be extended to deal with auctions that also include a common valuation component, as long as that component is publicly known and additively separable. For a model where takeovers are analyzed using private value auctions and where these issues are discussed in more detail see Burkart (1995).

the firm's corporate culture, some bidders will increase their valuations (the ones that have similar corporate cultures) whereas others will have their valuations decreased. Although the seller provides the information publicly, bidders' valuations after updating remain private information because valuations depend on idiosyncratic factors. As for internet auctions, their growing importance is evidenced by the great attention they are receiving from the public and the press as well as in the amount of value that is being traded in this way. Every day hundreds of thousands of objects change hands via internet auctions and many of these transactions match well with the model we are proposing,² in the sense that the seller often controls all the information that is going to be made public to potential buyers (by posting electronic images, providing text descriptions, etc.) but he knows very little about the specific preferences of the bidders who are going to participate in the auction.³

The main result of the paper is that in situations such as those described above the seller has incentives to release less information to bidders than would be efficient. The intuition behind this result is that, the auctioneer, by reducing the information held by the market, makes bidders more homogeneous and thus promotes fiercer competition. In a nutshell, ignorance promotes competition. We also show that when the market is more competitive (in the sense that the number of bidders is higher), the auctioneer releases more information. In particular, in a perfectly competitive market in which bidders get no rents, the auctioneer would provide the efficient level of information.

This paper sheds light on one trade-off in providing information in private value auctions. First, increasing information on the object to be sold improves the match between bidders' preferences and the characteristics of the object, and by doing so increases the willingness to pay of the winning bidder and the revenues of the auctioneer. Second, more information about the object increases the informational rents of the winning bidder, and this lowers the auctioneer's revenues. When the

²An empirical study of auctions on internet, Lucking-Reiley (2000), reports that online auctions currently trade billions of dollars' worth of goods per year and are growing a rate of more than 10% per month.

³According to Lucking-Reiley (2000), collectibles (antiques, stamps, coins, toys, trading cards, etc...) are the most traded goods. The collectibles fit well with the assumption of private valuations. Bidders preferences over object characteristics are different (depending on their particular taste and the collectibles they already own) and private information to bidders. As in the previous example, therefore, the information provided by the auctioneer is likely to affect asymmetrically to bidders' valuations.

an auctioneer decides how much information he provides to the market, he has to optimally balance these two opposing effects. Finally, the relative weight of these two opposing effects depends on the number of bidders. When the number of bidders is higher, bidders get lower informational rents, and the auctioneer will find it optimal to reveal more information to the market.

To illustrate our story, consider a very simple example. An auctioneer wants to sell an object using a second-price sealed-bid auction. There are two risk neutral bidders. Each bidder either likes the object and has a high valuation V_H or he does not like the object and has a low valuation V_L . The prior probability of the two events is $\frac{1}{2}$, so without additional information the expected valuation of each bidder is $\frac{1}{2}V_H + \frac{1}{2}V_L$. If the auctioneer discloses new information about the object, each bidder learns his true valuation, so that his posterior private valuation may be V_H or V_L . Hence the auctioneer gets a low expected price ($P_D = \frac{1}{4}V_H + \frac{3}{4}V_L$), the allocation of the object is efficient (the bidder with the highest valuation gets the object), and the bidders earn positive rents. If the auctioneer does not disclose any information about the object we have that, the expected price is higher ($P_{ND} = \frac{1}{2}V_H + \frac{1}{2}V_L$), the bidder with the lowest valuation can get the object and bidders earn zero rents. Therefore, the lack of information about the object promotes competition between bidders but can also lead to an inefficient allocation. If the number of bidders is 3, the auctioneer is indifferent between releasing the information or not, and if it is larger than 3, the auctioneer strictly prefers disclosing the information.

The remainder of the paper is organized as follows. In Section 2 we briefly review the related literature. The model is introduced in Section 3. In Section 4 we show how the bidders' valuations depend on the information the auctioneer releases concerning the object. Section 5 studies the auctioneer's information release and characterizes the efficient solution and the auctioneer's optimal strategy. We conclude by discussing the scope and implications of the model. All proofs are relegated to a technical appendix.

2 RELATED LITERATURE

This paper is related to the literature that studies endogenous information structures in auctions. Milgrom and Weber (1982) study a similar problem in which a seller wishes to sell a single object and has private information about its value. He must decide whether or not to reveal this information. They assume that the traders' valuations are affiliated (roughly, the valuation of the bidders and the seller are positively correlated) and provide the so-called linkage principle, which states that the seller's optimal strategy is to provide bidders with as much information as possible about the value of the object.⁴ The intuition behind this result is that by releasing all available information the seller increases on average the losing bids (reducing the winner's curse), and as a result increases the expected price of the object.⁵

The main difference between our model and the one analyzed by Milgrom and Weber (1982) is the way in which information affects bidders' valuations. In their model, the valuations of the seller and the bidders are positively correlated. This implies that all bidders react symmetrically to the information revealed by the seller. In contrast, we study a situation in which bidders' preferences about the characteristics of the object are heterogeneous, with the implication that increased information about the characteristics of the object will raise the valuation of some bidders and reduce the valuations of others.

In a recent paper, Bergemann and Pesendorfer (2001) study the optimal auction design problem when the auctioneer can set the accuracy with which bidders learn their valuation of the object in an independent valuations framework. In contrast to Milgrom and Weber (1982) and this paper, they allow the auctioneer to control the information structure of each individual bidder. In other words, the auctioneer is in the position of letting one bidder learn his valuation perfectly, while having a different bidder only get a rough estimate of his valuation. They show that the

⁴Ottaviani and Prat (2001) extend the logic of the linkage principle to the standard non-linear pricing monopoly problem. They show that the expected profits of a monopolist who sells to a single buyer cannot decrease by committing to publicly reveal information affiliated to the valuation of the buyer.

⁵Perry and Reny (1999) and de-Frutos and Rosenthal (1997) show that the linkage principle might fail in multi-unit auctions.

optimal information structures are partitions and that partitions must be asymmetric across bidders. They also find that the optimal selling strategy of the auctioneer can be implemented by a sequence of take-it-or-leave-it offers.

The focus of our analysis differs from Bergemann and Pesendorfer (2001) in that we consider it important to study situations in which the auctioneer's release of information is constrained to be public. In internet auctions, for instance, it is usually impossible to identify active bidders until the moment in which they are bidding, and this makes the provision of asymmetric information very difficult. In regulatory environments, legal restrictions often require the auctioneer to publicly release information in order to avoid favoritism or corruption.⁶ Even in situations in which the auctioneer is able to provide information asymmetrically, the bidders can trade and exchange information before bidding, undermining the desirable effects of information discrimination.

Given the different angle they take, Bergemann and Pesendorfer's methodology is very different from ours. They assume that the auctioneer can partition the support of the valuation distribution of each bidder and that bidders can privately learn the element of the partition in which his valuation lies. The auctioneer chooses the accuracy with which bidders learn their valuations by determining the fineness of the partitions. In contrast, we assume that bidders observe a public noisy signal of the real location of the object in the product space. The auctioneer determines the accuracy of the bidders' learning process by controlling the distribution of the noise. Our approach allows additional insights to be provided on the way in which the optimal release of information by the auctioneer changes with the number of bidders.

A number of other papers analyze endogenous information structures in auctions but they focus on the incentives of bidders to acquire information rather than on the optimal information provision by the auctioneer. Tan (1992) and Stegeman (1996) analyze the problem for private value auctions and Matthews (1984) and Persico (2001) study the pure common value auction and the case of affiliated valuations respectively.

⁶In practice, most public procurement processes make it mandatory for the sponsor to provide the same information to all potential contractors (see for example, the Green Book of Public Contracting in the European Union).

In a similar vein the literature on principal agent problems has also dealt with endogenous information structures. Sobel (1983), Cremer and Khalil (1992) and Cremer, Khalil and Rochet (1998) study incentive problems in which the agent decides whether or not to acquire private information. Our approach differs from these single agent models in that in the present paper the information acquisition process of the agents is controlled by the principal.

In terms of modeling choices our paper is closely related to Lewis and Sappington (1994) who focus on information acquisition by consumers in a monopoly market. The authors examine whether the monopolist should allow the consumers to acquire information about their tastes for his product. Improved private information enables the monopolist to charge higher prices to high-value buyers, but can also provide rents to the buyers. Most of their results are extreme, in the sense that the monopolist decides to provide either all the information or none. Aside from the differences in the information structures analyzed, our paper differs from Lewis and Sappington (1994) in that the price is set by an auction mechanism. Moscarini and Ottaviani (2001) obtain similar results to Lewis and Sappington (1994) in an oligopolistic environment. They study a situation in which two sellers compete for a single buyer who observes a private signal on the relative quality of their goods. As in Lewis and Sappington (1994) and in the present paper, they find that sellers may lose from the release of public information. Our paper differs from Moscarini and Ottaviani (2001) in that we focus on the imperfect competition on the demand side in an auction framework and we allow the seller to control the extent of buyers' private information. In the same line of research, Bergemann and Välimäki (1997) study information acquisition by consumers in a duopolistic market in which one firm introduces a new product whose value is learned by consumers through experimentation. The authors show that in equilibrium the sales path of the new product induces levels of experimentation that differ from the efficient ones. The intuition is that both firms want to speed up the learning process in the early stages in order to obtain rents due to product differentiation. In this paper, the auctioneer induces a suboptimal learning process to reduce bidders' differentiation and consequently bidders' rents.

3 THE MODEL

Consider an auctioneer who plans to sell an object. The object can have different characteristics that can be summarized by a location in a product space, $x^* \in \Phi$, where the product space Φ is a circle of perimeter one. There are N risk-neutral bidders, $i = \{1, \dots, N\}$. The location of each bidder, x_i , which is private information, is uniformly distributed on the circle Φ . Each bidder has a preferred specification for the characteristics of the object, its location $x_i \in \Phi$, and his object valuation for an arbitrary location x is $v_i(x) = V - \beta(x - x_i)^2$ where β is a measure of the transportation cost of the bidders with respect to the product space. The market is initially uncertain about the exact location of the object in the product space so that, ex ante, x^* is distributed according to the uniform distribution on the circle.

Before bidders make their offers, the auctioneer can provide information about the object's attributes to the bidders and alter their expected valuations. We model bidders' acquisition of information through the realization of a public noisy signal of the location of the object in the product space, denoted by $\hat{x} \in \Phi$. The auctioneer controls the accuracy of this signal by deciding how much information to provide to bidders. Obtaining and transmitting information is costly.⁷ Let $\delta \in [0, \infty)$ denote the cost incurred by the auctioneer in providing information, where a higher δ implies a more accurate signal \hat{x} .

The relationship between the location of the object x^* , the public signal \hat{x} and the cost of information δ is the following: the public signal is $\hat{x} = x^* + \varepsilon$, where the noise ε is distributed over $[-\frac{1}{2}, \frac{1}{2}]$ according to the distribution function $G(\varepsilon|\delta)$. We make the following assumptions about this distribution:

ASSUMPTION 1 *The density function associated with $G(\varepsilon|\delta)$ is symmetric and centered at 0.*

ASSUMPTION 2 *When $\delta = 0$, $G(\varepsilon|\delta)$ is equal to the uniform distribution on $[-\frac{1}{2}, \frac{1}{2}]$. When $\delta \rightarrow \infty$,*

⁷This cost might include advertisement, research about product characteristics, providing testing opportunities or samples to potential bidders, etc.

the $G(\varepsilon|\delta)$ converges to⁸:

$$G(\varepsilon|\infty) = \begin{cases} 0 & \text{if } \varepsilon < 0 \\ 1 & \text{otherwise} \end{cases}$$

ASSUMPTION 3 $G(\varepsilon|\delta)$ is differentiable and decreasing (increasing) in δ for all ε lower (greater) than 0; that is, $\frac{\partial G(\varepsilon|\delta)}{\partial \delta} < 0$ for all $\varepsilon \in (-\frac{1}{2}, 0)$, and $\frac{\partial G(\varepsilon|\delta)}{\partial \delta} > 0$ for all $\varepsilon \in (0, \frac{1}{2})$.⁹

[Figure 1 around here]

Given these assumptions the public signal \hat{x} is also distributed uniformly around the circle, and by Assumption 1 it is an unbiased estimator of the location of the object in the product space. Assumption 2 implies that when $\delta = 0$ the signal provides no information, whereas when $\delta = \infty$ the signal is fully informative. Assumption 3 implies that the variance of the noise decreases with δ .¹⁰ Thus, when the cost of the information provided by the auctioneer, δ , is high, the signal \hat{x} is more informative about the location of the object; this means that a more costly signal is more informative, so that we will occasionally refer to δ as the amount of information of a public signal with cost δ .

Once bidders observe the information released by the auctioneer (the public signal \hat{x}), the distribution of x^* is no longer uniform. Instead it is distributed on the circle according to a

⁸That is, as δ goes to infinity, the probability that ε will be equal to zero converges to 1.

⁹The truncated normal distribution defined on the interval $[-\frac{1}{2}, \frac{1}{2}]$ when the underlying normal distribution has mean $\mu = 0$ and variance $\sigma = \frac{1}{\delta}$, is an example of a distribution that is consistent with these three assumptions.

$$G(\varepsilon|\delta) = \frac{\int_{-\frac{1}{2}}^{\varepsilon} \exp\{-s^2\delta\} ds}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \exp\{-s^2\delta\} ds}.$$

¹⁰An assumption alternative to Assumption 3 is:

Assumption 3' If $\delta > \delta'$, we can order the distribution functions $G(\varepsilon|\delta)$ and $G(\varepsilon|\delta')$ in the sense of first order stochastic dominance: $G(\varepsilon|\delta) \leq G(\varepsilon|\delta') \forall \varepsilon \in [-\frac{1}{2}, 0]$, $G(\varepsilon|\delta) \geq G(\varepsilon|\delta') \forall \varepsilon \in [0, \frac{1}{2}]$.

An example of a distribution that is consistent with this assumption is the uniform distribution on an interval decreasing in δ , $[-\frac{1}{2(1+\delta)}, \frac{1}{2(1+\delta)}]$:

$$G(\varepsilon|\delta) = \begin{cases} 0 & \text{if } \varepsilon < -\frac{1}{2(1+\delta)} \\ (1+\delta)\varepsilon + \frac{1}{2} & \text{if } \varepsilon \in [-\frac{1}{2(1+\delta)}, \frac{1}{2(1+\delta)}] \\ 1 & \text{if } \varepsilon > \frac{1}{2(1+\delta)} \end{cases}$$

Under Assumption 3', using the techniques of Milgrom and Shannon (1994) we would obtain weakly monotonic comparative results. Assumption 3 on the other hand lead us to strictly monotonic results.

posterior distribution $F(x|\delta)$ that depends on the realization of the signal \hat{x} and the amount δ of information provided.¹¹

After the auctioneer has released the information, the awarding process takes place. Bidders, using the posterior distribution of x^* , update their expected valuations of the object and submit their offers to the auctioneer. The auctioneer sells the object using a second-price sealed-bid auction. For simplicity we abstract from reserve prices and assume that the object is always sold. Summarizing, the time sequence of the model is as follows:

1. The auctioneer, knowing the number of bidders, N , but not their preferences (locations) decides how much information to provide to the market by choosing δ .
2. Given δ , bidders receive a public signal \hat{x} which is used as an initial estimate of the location of the object in the product space.
3. According to δ and \hat{x} , bidders update their valuations of the object.
4. The second-price sealed-bid auction takes place.

We have chosen the second-price sealed-bid auction for our model on the basis of computational simplicity and ease of presentation. As will be clear in the following, our setting satisfies the conditions of the revenue equivalence theorem which states that any auction mechanism in which the object is always awarded to the buyer with the highest valuation and where any bidder with the lowest valuation obtains zero surplus, yields the same expected revenue to the auctioneer. Thus, all “standard” auctions (second-price sealed-bid, first-price sealed-bid, oral ascending (English) or oral descending (Dutch)) and many non-standard auctions such as an “all-pay” auction would yield the same expected revenue to the auctioneer, bidders would make the same expected payments as a function of their valuations and, as a consequence, the same results would be obtained.

¹¹For notational convenience, we take the location of the public signal as the origin of the circle, and we define $F(x|\delta)$ on the interval $[\hat{x} - \frac{1}{2}, \hat{x} + \frac{1}{2}]$. Given the above assumptions on the noise distribution, $F(x|\delta)$ presents the following characteristics: (i) The density function associated to $F(x|\delta)$ is symmetric and centered at \hat{x} . (ii) When $\delta = 0$, $F(x|\delta)$ is equal to the uniform distribution on $[\hat{x} - \frac{1}{2}, \hat{x} + \frac{1}{2}]$. When $\delta \rightarrow \infty$, $F(x|\delta)$ converges to:
$$F(x|\infty) = \begin{cases} 0 & \text{if } x < \hat{x} \\ 1 & \text{otherwise} \end{cases}$$
 .(iii) $F(x|\delta)$ is decreasing in δ for all x lower than \hat{x} and increasing in δ for all x greater than \hat{x} .

We characterize the Perfect Bayesian equilibrium starting from the auction and moving backwards. In the next section we study how bidders update their valuation given δ and \hat{x} , and the outcome of the auction. Section 5 focuses on the auctioneer's decision about how much information to provide to the bidders. First, we characterize the efficient solution of this problem and then we characterize the auctioneer's optimal information release. Finally, we compare both solutions.

4 ENDOGENOUS BIDDERS VALUATIONS

The goal of this section is to study how the expected valuation of the bidders depends on the distance between their locations and the public signal \hat{x} and how the distribution of expected valuations changes when bidders update their valuations after observing \hat{x} .

Given the information provided by the auctioneer, bidders make an initial estimation of the location of the object in the product space (they receive a public signal \hat{x}) and update their valuations. Let $V(x_i, \delta)$ be the expected valuation of the object for a bidder located at x_i , when the amount of information is δ :

$$V(x_i, \delta) = E_{x^*} \{v_i(x)\}.$$

The following lemma shows us how the expected valuation of the object depends on the distance between the bidder's location, x_i , and the public signal, \hat{x} .

LEMMA 1 If $\delta > 0$, then the expected valuation of the object is decreasing in the distance between the bidder's location, x_i , and the public signal, \hat{x} . If $\delta = 0$, the expected valuation is independent of bidder's location and the public signal.

The intuition behind this result is the following. The valuation of the object is decreasing in the distance between the most preferred location of the bidder and the location of the object x^* . Since signal \hat{x} is an unbiased estimator of x^* , the expected valuation of the object is decreasing in the distance between the bidder's location and \hat{x} . On the other hand, if the auctioneer does not invest in providing information to the bidders, i.e. $\delta = 0$, the object can be located with the

same probability on any arbitrary place in the circle, implying that the expected valuation of any bidder is the same.

Let x_1 be the location of the bidder closest to \hat{x} . An immediate corollary of the previous Lemma characterizes the bidder with the highest valuation.

COROLLARY 1 *The highest expected valuation is the valuation of the closest bidder to \hat{x} , $V(x_1, \delta)$.*

Notice that the bidder closest to \hat{x} may turn out not to be the bidder with the highest ex-post valuation of the object. It can be shown that the probability of this event is decreasing in the amount of information provided by the auctioneer.

4.1 Bidders' valuations distribution

Once the auctioneer's information is revealed and bidders update their valuations, we can regard our model as a standard private value auction model. Each bidder is characterized by his expected valuation v_i , which is private information to him, and it is common knowledge that each bidder's valuation v_i is an independent realization of a continuous random variable distributed over $[\underline{v}, \bar{v}]$ according to $H(v, \delta)$ where $\underline{v} = V(\hat{x} + \frac{1}{2}, \delta)$ and $\bar{v} = V(\hat{x}, \delta)$.¹² Given Lemma 1 the expected valuation of the object is a continuous and strictly decreasing function of the distance between a bidder's location, x_i , and \hat{x} , so that the distribution of valuations $H(v, \delta)$ can be obtained by computing the distribution of the distances between bidders' locations and \hat{x} using a simple change of variable.¹³

To finish this section, we present the outcome of the awarding process.

LEMMA 2 *The bidder closest to \hat{x} wins the second-price sealed-bid auction at a price equal to the expected valuation of the second closest bidder, $V(x_2, \delta)$.*

¹²Notice that the bidders' valuations are independently distributed and privately known, since their locations were independently distributed and privately known.

¹³Notice that given that the bidder's location are distributed according to a uniform distribution over the circle, the distance between the location of the bidders and \hat{x} is also distributed according with to a uniform distribution over the interval $[0, \frac{1}{2}]$. Further details and additional characterization of $H(v, \delta)$ are provided in the appendix.

5 INFORMATION RELEASE

In this section, we study the amount of information about product characteristics that the auctioneer will provide to the bidders. We start by characterizing the efficient information release.

5.1 Efficient Information Release

The objective of this section is to characterize the efficient information release, which is that which maximizes the total surplus, namely the sum of the auctioneer's revenue and the utility of the winning bidder (the bidder with the highest expected valuation).

The expected valuation of the winning bidder at the initial stage, $V(\delta)$, is a function which only depends on the amount of information:

$$V(\delta) = E_{x_1, x^*} \{V(x_1, \delta)\}.$$

The next result characterizes the relationship between this expected winning valuation and the amount of information provided by the auctioneer.

LEMMA 3 *The expected valuation of the winning bidder, $V(\delta)$, is increasing in the amount of information, δ .*

Lemma 3 rests on the fact that the greater the information provided to bidders, the better the matching between the object and the preferences of the winning bidder.

Given the above, the efficient information release, δ^E , arises from the optimal trade off between increasing the expected valuation of the winning bidder and the cost of providing information to the market:

$$\delta^E \in \operatorname{argmax}_{\delta} E_{x_1, x^*} \{V(x_1, \delta) - \delta\}. \quad (1)$$

First observe that given that $V(x_1, \delta)$ is bounded from above, δ^E has to be finite. In the following we will assume that $\delta^E > 0$, an assumption which is justified if the cost of providing very basic information about the object is sufficiently small. Then we have the following:

PROPOSITION 2 *The efficient amount of information, δ^E , is increasing both in the number of bidders, N , and in the transportation cost parameter, β .*

The valuation of the winning bidder depends on the distance between his location and the actual location of the object in the product space. When the number of bidders increases, the expected distance between the initial estimate of the object's location \hat{x} and the location of the winning bidder decreases. As a result, the incentives to make \hat{x} closer to the actual location of the object x^* also increase. Using a similar argument, if the parameter β increases, the incentive to reduce the distance between the location of the winning bidder and the actual location of the object increases (the match between the object and the winning bidder's preferences becomes more important). The only way to ensure an appropriate match is to reduce the distance between the actual location of the object and the initial bidders' estimate and this, in turn, can only be accomplished by increasing the information provided to bidders.¹⁴

5.2 *The Auctioneer's Optimal Information Release*

The objective of this subsection is to characterize the information release that maximizes the auctioneer's revenue. First, we study how the expected rents of the winning bidder depend on the amount of information provided by the auctioneer. The informational rents of the winning bidder are the difference between his valuation and the valuation of the second closest bidder (which is the price of the object)

$$\Pi_w(\delta) = V(x_1, \delta) - V(x_2, \delta).$$

PROPOSITION 3 *The expected informational rents of the winning bidder are increasing in δ .*

This is an important result: when the amount of information provided to the bidders about object characteristics is higher, the value of private information on preferences over object char-

¹⁴Notice that we have not imposed assumptions on the convexity of the problem and therefore cannot guarantee that the problem is concave. We use the techniques of Edlin and Shannon (1998), which allow us to get comparative statics results in non-convex problems, as long as the cross derivative conditions are globally satisfied by the problem, a condition that is satisfied in our case.

acteristics is higher. The implication of this result is that it can be optimal for the auctioneer to restrict the information provided to the market in order to control bidders' rents.

The auctioneer's optimal information release, δ^A , maximizes the difference between the expected price and the cost of providing more information to the bidders.

$$\delta^A \in \operatorname{argmax}_{\delta} E_{x_2, x^*} \{V(x_2, \delta) - \delta\} \quad (2)$$

By comparing (1) and (2) it is easy to see that the expected revenue of the auctioneer does not depend on the location of the winning bidder (as is the case in the total surplus), rather on the location of the bidder which is the second closest to the estimate of the object location, \hat{x} . Apart from this fact, the auctioneer's problem is identical to the total surplus maximization problem, and the intuition behind the results presented in the following Proposition is the same as in Proposition 2.

PROPOSITION 4 *The optimal amount of information δ^A provided by the auctioneer is increasing both in the number of the bidders, N , and in the transportation cost parameter, β .*

We assume that $\delta^A > 0$, as in Proposition 2. The following theorem presents the main result of the paper.

THEOREM 5 *The auctioneer provides less information to bidders than would be efficient, $\delta^A < \delta^E$. The difference between the efficient information release and the equilibrium information release converges to 0 as the number of bidders goes to infinity.*

As was remarked above, the auctioneer's problem would be equivalent to the total surplus maximization problem if in the latter we considered the second closest bidder instead of the closest bidder. Using this, it is then easy to see the intuition of Theorem 5. From Proposition 2 we know that the larger the number of bidders, the closer the location of the winning bidder to \hat{x} , and the greater the incentives to provide more information to the market. Using the same argument, if we take the second closest bidder instead of the closest bidder, there should be less

incentives to provide information to the market.¹⁵

To better understand the result one can rewrite the problem of the auctioneer as

$$\delta^A \in \operatorname{argmax}_{\delta} E_{x_1, x_2} \{V(x_1, \delta) - \delta - \Pi_w(\delta)\} \quad (3)$$

This formulation clarifies an important trade-off when providing information to the market. On the one hand, when the auctioneer provides more information, the efficiency of the auction process goes up ($E_{x_1} \{V(x_1, \delta)\}$ is increasing in δ from Lemma 3). This follows because the winning bidder is more likely to be the bidder with the highest ex-post valuation of the object. On the other hand, the increase in information also raises the informational rents of the winning bidder ($\Pi_w(\delta)$ is increasing in δ from Proposition 3). The optimal balance of these two opposing effects leads the auctioneer to provide less information than would be efficient.¹⁶ In other words, the auctioneer will restrict the information released to the market in order to make the potential bidders more homogeneous, with the underlying goal of intensifying competition and increasing his expected revenue.

Finally, when the number of bidders goes to infinity, the rents of the closest bidder converges to 0 because the expected distance to the second closest bidder also converges to 0. In such a case, the auctioneer's trade off between reducing the bidders' rents and increasing the auction efficiency is eliminated as can be seen from the fact that $E_{x_1, x_2} \{V(x_1, \delta) - V(x_2, \delta)\}$ goes to 0.

¹⁵In the paper it is assumed that the provision of some information is socially efficient. We could also consider the case in which it is efficient not to provide information at all. In this case, it can be shown that it would still be optimal for the auctioneer not to release any information. I thank a referee for pointing out this extension of the result.

¹⁶The results of Theorem 5 are obtained under the assumption that the object is always sold. The seller could improve his outcome by introducing a reserve price in the auction. Clearly, this case would require a more detailed analysis, but we believe that the same type of result would also hold. The idea goes as follows. We have shown that providing information to buyers has the drawback of increasing their informational rents; however, a reserve price could be used by the seller to reduce these rents. On the one hand, this implies that a seller using an optimal reserve price would have incentives to provide more information than otherwise. On the other hand, since optimal reserve prices reduce but do not eliminate informational rents, it seems reasonable to conjecture that the seller would not have incentives to provide the efficient amount of information.

6 CONCLUSIONS

Most of the research in auction theory presumes that the information held by bidders is exogenous. In contrast, we have analyzed a model in which the auctioneer has private information about the characteristics of the object to be sold and can control how much information to reveal to bidders. We have shown how the auctioneer provides less information than would be efficient since by doing so he reduces the informational rents of the winning bidder. Moreover, we have identified some factors that may influence the amount of information that the auctioneer will provide to the bidders. In particular, we have shown that a more competitive market will induce the auctioneer to provide more information. In the limit, when the number of bidders goes to infinity, the auctioneer optimally releases the efficient amount of information.

One important difference between our model and the standard auction models is that we endogenize the distribution of bidder valuations. Rather than taking the distribution of valuations as exogenous, we have allowed bidders to have exogenous privately known preferences but have specified how the information provided by the auctioneer on object characteristics interacts with preferences to generate ex-post distributions of private valuations. We do not treat the problem in full generality. In particular, first we assume that the information provided by the auctioneer is related only to the location of the good in the product space, and second that the auctioneer provides information to all agents symmetrically.

The symmetry imposed on the product space and on the distribution of the bidders' preferences implies that the auctioneer has no ex-ante motives to distort information, as all locations give him the same expected revenue. This allows us to ignore all problems related to the strategic revelation of information without assuming that the auctioneer can commit to not censoring the information (only providing favorable information), as in Milgrom and Weber (1982) and Bergemann and Pesendorfer (2001). By isolating the decision of how much information to reveal we have found that there are two factors that determine the optimal provision of information: (i) improved information increases the efficiency of the auction; (ii) improved information also

increases the informational rents of the winning bidder. These two effects represent opposing forces for the auctioneer: improved efficiency raises revenues while increased informational rents reduce revenues. As the number of bidders rises, the first effect is increased and the second effect is reduced. Hence, the auctioneer provides more information, so competition increases the information provided. Further research is needed to extend these results to more realistic environments with non-homogeneous product spaces and more general preference distributions, where the strategic revelation of information will have to be incorporated into the analysis.

Finally, let me point out that the idea behind this paper can be used to explain the presence of cost overruns in public works. A companion paper, Ganuza (2000), studies cost overruns in a procurement model in which a sponsor wants to undertake a public work that can have different designs.¹⁷ The design space is a circle and N potential contractors compete to be awarded the contract. Like bidders in the present paper, the contractors are horizontally differentiated, in this case according to their specialization in a specific design (their location), and they face a cost of realizing an arbitrary design that is increasing with its distance from its location. Unlike the present paper, however, the sponsor does not know his preferred design. Prior to the awarding process the sponsor decides how much to invest in specifying an initial design (or blueprint) for the public work and this decision becomes public information. Similar to the present paper, on the other hand, the initial design can be considered as a noisy signal of the optimal design. The sponsor auctions the realization of this initial design and the winning firm signs a contract to undertake this initial design. During the construction of the project, the sponsor and the firm learn the optimal design, and renegotiate the initial contract. Cost overruns, i.e. the difference between the final price of the project and the procurement price are a consequence of this renegotiation.

In such a framework, when the sponsor invests more in the specification of the initial design, the initial design is more likely to be closer to the optimal one, substantial reforms are less likely to be necessary, and cost overruns are less likely to be sizable. As is often claimed, a low investment

¹⁷The companion working paper is entitled “Competition and Cost Overruns. Optimal Misspecification of Procurement Contracts”. This working paper is available from the author upon request and can be downloaded at <http://www.econ.upf.es/cgi-bin/onepaper?471>.

in the initial design specification is likely to lead to negotiating significant changes and therefore to high cost overruns. However, investing in design specification is shown to have a drawback, as it increases the rents of the winning firm. This implies that the optimal strategy of the sponsor is to underinvest in design specification so as to make significant cost overruns likely. The key idea is the same as the one in this paper: by reducing design specification (information about the optimal design) the sponsor promotes fiercer competition among contractors. As in this paper, an increase in the number of potential contractors increases the level of optimal specification and, in a perfectly competitive market, no such misspecification occurs.¹⁸

7 APPENDIX

As a convention and without loss of generality we are going to consider in the appendix that $\hat{x} = 0$ and that $x_i, x_1, x_2 \in [0, \frac{1}{2}]$. Notice that with this convention the location of the bidders x_i, x_1, x_2 are also their distance to \hat{x} . We need to state some preliminary facts before we start with the proofs of the results.

Let $G_{x_1}(x, N)$ and $G_{x_i}(x, N)$ be the distributions of the expected distance between the public signal \hat{x} and (i) the closest bidder and (ii) the bidder i closest to \hat{x} . These distributions do not depend on \hat{x} and it can be shown that $\frac{\partial G_{x_1}(x, N)}{\partial N} < 0 \forall x \in (0, \frac{1}{2})$. These distributions are ordered in a strict first order stochastic dominance sense: $G_{x_1}(x, N) > G_{x_i}(x, N)$, for all $x \in (0, \frac{1}{2})$.

Proof of Lemma 1: The expected valuation of an arbitrary bidder x_i , given that $\hat{x} = 0$ and the amount of information is δ , is

$$V(x_i, \delta) = E_{x^*} \{V - \beta(x_i - x^*)^2 | \delta\}.$$

Since x^* is distributed on $[-\frac{1}{2}, \frac{1}{2}]$ according to $F(x|\delta)$, this expectation is

$$V(x_i, \delta) = V - \int_{-\frac{1}{2}}^{\frac{1}{2}} \beta \min\{(x_i - s)^2, (1 - |x_i - s|)^2\} f(s|\delta) ds.$$

Notice that, due to the fact that the product space is a circle, there are two distances between x_i

¹⁸These results do not depend on specific design renegotiation procedures, since the incumbent rents of the renegotiation are discounted by the potential contractors in the auction. See Ganuza (2000) for details.

and x^* and we have to consider only the shortest length arc.

$$\begin{aligned} V(x_i, \delta) &= V - \int_0^{\frac{1}{2}-x_i} \beta(x_i - s)^2 f(s|\delta) ds - \int_{\frac{1}{2}-x_i}^{\frac{1}{2}} \beta(x_i - s)^2 f(s|\delta) ds - \\ &\quad - \int_{-\frac{1}{2}+x_i}^0 \beta(x_i - s)^2 f(s|\delta) ds - \int_{-\frac{1}{2}}^{-\frac{1}{2}+x_i} \beta(1 - x_i + s)^2 f(s|\delta) ds. \end{aligned}$$

By using the symmetry of $F(x|\delta)$ we get

$$\begin{aligned} V(x_i, \delta) &= V - \int_0^{\frac{1}{2}-x_i} \beta((x_i - s)^2 + (x_i + s)^2) f(s|\delta) ds \\ &\quad - \int_{\frac{1}{2}-x_i}^{\frac{1}{2}} \beta((x_i - s)^2 + (1 - x_i - s)^2) f(s|\delta) ds \\ V(x_i, \delta) &= V - \int_0^{\frac{1}{2}} 2\beta(x_i^2 + s^2) f(s|\delta) ds - \int_{\frac{1}{2}-x_i}^{\frac{1}{2}} \beta(1 - 2(x_i + s)) f(s|\delta) ds. \end{aligned}$$

Integrating by parts the second term we get

$$V(x_i, \delta) = V - \beta \left(x_i^2 + \int_0^{\frac{1}{2}} 2s^2 f(s|\delta) ds - 2x_i + \int_{\frac{1}{2}-x_i}^{\frac{1}{2}} 2F(s|\delta) ds \right).$$

It is interesting to see the special cases $\delta = 0$ and $\delta = \infty$. We have that

$$V(x_i, 0) = V - \beta \int_0^{\frac{1}{2}} 2s^2 ds.$$

If the public signal $\hat{x} = 0$ is not informative, the bidder's expected valuation does not depend on his location. On the other hand

$$V(x_i, \infty) = V - \beta x_i^2,$$

when there is perfect information about the object's location, so $\delta = \infty$. Here, the expected valuation only depends on the location of the bidder. For interior cases, we differentiate $V(x_i, \delta)$ with respect to x_i

$$\frac{\partial V(x_i, \delta)}{\partial x_i} = -\beta \left(2x_i - 2 + 2F\left(\frac{1}{2} - x_i|\delta\right) \right).$$

We have $\frac{\partial V(x_i, 0)}{\partial x_i} = 0$. Furthermore, $F(\frac{1}{2} - x_i|\delta)$ is increasing in δ , and therefore $\frac{\partial V(x_i, \delta)}{\partial x_i} < 0$ for all $\delta > 0$. The valuation of the object is therefore decreasing in the distance between the public signal \hat{x} and the bidder's location. *Q.E.D.*

Proof of Corollary 1: Immediate from Lemma 1. *Q.E.D.*

Characterization of the distribution of valuations $H(v, \delta)$: Let $s(v, \delta)$ be the inverse function of $V(x_i, \delta)$ when $x_i \in [0, \frac{1}{2}]$. Thus, the density function $h(v, \delta)$ is

$$h(v, \delta) = 2 \left| \frac{ds(v, \delta)}{dv} \right|$$

where, we are using the fact that the distance between an arbitrary bidder and \hat{x} is distributed according to a uniform distribution on the interval $[0, \frac{1}{2}]$. As the distance varies over the interval $[0, \frac{1}{2}]$, and $V(x_i, \delta)$ is decreasing when $x_i \in [0, \frac{1}{2}]$, it is found that the valuations vary over $[\underline{v}, \bar{v}]$ where $\underline{v} = V(\frac{1}{2}, \delta)$ and $\bar{v} = V(0, \delta)$. As $F(x_i|\delta)$ is increasing in δ we can show that $\underline{v} = V(\frac{1}{2}, \delta) = V - \beta \left(\int_0^{\frac{1}{2}} (2 - 4s)F(s|\delta)ds - \frac{1}{4} \right)$ is decreasing in δ and $\bar{v} = V(0, \delta) = V - \beta \left(-\int_0^{\frac{1}{2}} 4sF(s|\delta)ds + \frac{1}{4} \right)$ is increasing in δ . When $\delta = 0$, $F(x_i|0)$ is uniform, and \underline{v} and \bar{v} coincide. Thus, the larger is δ , the more spread the distribution, and the larger the interval $[\underline{v}, \bar{v}]$.

Then, the distribution function $H(v, \delta)$ is

$$H(v, \delta) = \int_{\underline{v}}^v 2 \left| \frac{ds(v, \delta)}{dv} \right| dv = 1 - U(s(v, \delta)) = 1 - 2s(v, \delta)$$

where $U(\cdot)$ is the uniform distribution on the interval $[0, \frac{1}{2}]$.

Proof of Lemma 2: Immediate from Lemma 1. *Q.E.D.*

Proof of Lemma 3: The expected highest valuation, given that the winning bidder is the closest bidder to the public signal \hat{x} , is

$$V(\delta) = E_{x_1} \{V(x_1, \delta)\} = E_{x^*, x_1} \{V - \beta(x_1 - x^*)^2|\delta\}$$

Therefore, to prove the lemma we have to show that $E_{x^*, x_1} \{(x^* - x_1)^2|\delta\}$ is decreasing in δ . First, we analyze the sum of the expected quadratic distance between the real location of the object and all the bidders. Let A_i be the expected quadratic distance between the real location of the object and the bidder which is the bidder i closest to the public signal \hat{x} .

$$\begin{aligned} \sum_{i=1}^N A_i &= E_{x^*, x_1} \{(x^* - x_1)^2|\delta\} \\ &\quad + E_{x^*, x_2} \{(x^* - x_2)^2|\delta\} + \cdots + E_{x^*, x_N} \{(x^* - x_N)^2|\delta\}. \end{aligned}$$

Rearranging terms we get

$$\sum_{i=1}^N A_i = NA_1 + \sum_{i=2}^N A_i - A_1.$$

It is clear that this sum does not depend on δ since the relative position of the bidders is not important when we are adding all the distances. Therefore, the derivative of this sum respect to δ has to be 0, so

$$N \frac{\partial A_1}{\partial \delta} + \sum_{i=2}^N \frac{\partial (A_i - A_1)}{\partial \delta} = 0.$$

The next step is to show that $\frac{\partial (A_i - A_1)}{\partial \delta} > 0$ for every $i \neq 1$. Using similar computations to those in the proof of Lemma 1 we get¹⁹

$$A_i - A_1 = \int_0^{\frac{1}{2}} \left(z^2 - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta) ds \right) (g_{x_i}(z, N) - g_{x_1}(z, N)) dz.$$

Integrating by parts

$$\begin{aligned} A_i - A_1 &= \left[\left(z^2 - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta) ds \right) (G_{x_i}(z, N) - G_{x_1}(z, N)) \right]_0^{\frac{1}{2}} + \\ &\quad - \int_0^{\frac{1}{2}} \left(2z - 2 + 2F\left(\frac{1}{2} - z|\delta\right) \right) (G_{x_i}(z, N) - G_{x_1}(z, N)) dz. \end{aligned}$$

Finally, taking the derivatives with respect to δ

$$\frac{\partial (A_i - A_1)}{\partial \delta} = - \int_0^{\frac{1}{2}} 2 \frac{\partial F(\frac{1}{2} - z|\delta)}{\partial \delta} (G_{x_i}(z, N) - G_{x_1}(z, N)) dz > 0$$

since $\frac{\partial F(s|\delta)}{\partial \delta} > 0$ and $G_{x_1}(z, N) > G_{x_i}(z, N) \forall z \in (0, \frac{1}{2})$. But given that the derivative of the sum is 0, and given that $\frac{\partial (A_i - A_1)}{\partial \delta} > 0$ for every $i \neq 1$, this implies that $\frac{\partial A_1}{\partial \delta} < 0$, which concludes the proof. *Q.E.D.*

Proof of Proposition 2: We are going to use a result of Edlin and Shannon (1998), which allows us to obtain strictly monotonic static comparative results without making assumptions on the concavity of the distribution functions.

THEOREM 6 (EDLIN AND SHANNON (1998)) *Let $S \subset \mathfrak{R}$, $f : \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$, $y^* \in \operatorname{argmax}_{y \in S} f(y, t^*)$ and $y' \in \operatorname{argmax}_{y \in S} f(y, t')$. Suppose that f is C^1 and has increasing marginal returns, and that $y^* \in \operatorname{int} S$. Then $y' > y^*$ if $t' > t^*$, and $y' < y^*$ if $t' < t^*$.*

¹⁹Notice that x_x and x^* are independent variables, and we do not need to specify the joint distribution.

We have to check that we can apply this theorem to our decision problem. Our problem is:

$$\begin{aligned} \max_{\delta} E_{x_1} \{V(x_1, \delta) - \delta\} \\ = \int_0^{\frac{1}{2}} \left(V - \beta \left(z^2 + \int_0^{\frac{1}{2}} 2s^2 f(s|\delta) ds - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta) ds \right) \right) \\ \times g_{x_1}(z, N) dz - \delta. \end{aligned}$$

We define the objective function $f(y, t)$ as

$$\begin{aligned} f(\delta, N) = \int_0^{\frac{1}{2}} \left(V - \beta \left(z^2 + \int_0^{\frac{1}{2}} 2s^2 f(s|\delta) ds - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta) ds \right) \right) \\ \times g_{x_1}(z, N) dz - \delta. \end{aligned}$$

where $y = \delta, t = N,$ and $S = \mathfrak{R}^+ \cup 0$. Therefore, the only condition that we have to check is that $f(y, t)$ has increasing marginal returns, so that $\frac{\partial f}{\partial y}$ is increasing in t .

To verify this condition, we compute the cross derivative $\frac{\partial^2 f}{\partial N \partial \delta}$. First, from differentiating with respect to δ , we get

$$\frac{\partial f}{\partial \delta} = -\beta \int_0^{\frac{1}{2}} \left(\int_0^{\frac{1}{2}} 2s^2 \frac{\partial f(s|\delta)}{\partial \delta} ds + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2 \left(\frac{\partial F(s|\delta)}{\partial \delta} \right) ds \right) g_{x_1}(z, N) dz - 1.$$

Integrating by parts and differentiating $\frac{\partial f}{\partial \delta}$ with respect to N we get

$$\frac{\partial^2 f}{\partial N \partial \delta} = \beta \int_0^{\frac{1}{2}} 2 \left(\frac{\partial F(\frac{1}{2} - z|\delta)}{\partial \delta} \right) \left(\frac{\partial G_{x_1}(z, N)}{\partial N} \right) dz.$$

Since $\frac{\partial F(\frac{1}{2} - z|\delta)}{\partial \delta} > 0$ and $\frac{\partial G_{x_1}(z, N)}{\partial N} > 0$ we have that the whole expression is positive, and hence $f(\delta, N)$ has increasing marginal returns. Therefore, applying Theorem 6 we conclude that the efficient amount of information δ^E is increasing in the number of bidders N .

We use the same argument for β . Thus, we compute the cross derivative

$$\frac{\partial^2 f}{\partial \beta \partial \delta} = - \int_0^{\frac{1}{2}} \left(\int_0^{\frac{1}{2}} 2s^2 \frac{\partial f(s|\delta)}{\partial \delta} ds + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2 \left(\frac{\partial F(s|\delta)}{\partial \delta} \right) ds \right) g_{x_1}(z, N) dz$$

This expression is positive, since by lemma 3 we know that $E_{x_1}\{V(x_1, \delta)\}$ is increasing in δ , and this implies that

$$\int_0^{\frac{1}{2}} \left(\int_0^{\frac{1}{2}} 2s^2 \frac{\partial f(s|\delta)}{\partial \delta} ds + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2 \left(\frac{\partial F(s|\delta)}{\partial \delta} \right) ds \right) g_{x_1}(z, N) dz$$

is negative. Then, applying Theorem 6, we conclude that efficient amount of information δ^E is increasing in β . *Q.E.D.*

Proof of Proposition 3: The informational rents of the winning bidder are the difference between his valuation and the expected valuation of the second closest bidder to the public signal \hat{x} .

$$\begin{aligned}\Pi_w(\delta) &= E_{x_1}\{V(x_1, \delta)\} - E_{x_2}\{V(x_2, \delta)\} \\ &= \int_0^{\frac{1}{2}} -\beta \left(z^2 - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta)ds \right) (g_{x_1}(z, N) - g_{x_2}(z, N)) dz\end{aligned}$$

We integrate this expression by parts to get

$$\begin{aligned}& \left[-\beta \left(z^2 - 2z + \int_{\frac{1}{2}-z}^{\frac{1}{2}} 2F(s|\delta)ds \right) (G_{x_1}(z, N) - G_{x_2}(z, N)) \right]_0^{\frac{1}{2}} \\ & + \int_0^{\frac{1}{2}} \left(\beta(2z - 2 + 2F(\frac{1}{2} - z|\delta)) \right) (G_{x_1}(z, N) - G_{x_2}(z, N)) dz\end{aligned}$$

Differentiating with respect to δ we get

$$\frac{\partial \Pi_w(\delta)}{\partial \delta} = \int_0^{\frac{1}{2}} \left(2\beta \frac{\partial F(\frac{1}{2} - z|\delta)}{\partial \delta} \right) (G_{x_1}(z, N) - G_{x_2}(z, N)) dz$$

This expression is positive, since $\frac{\partial F(\frac{1}{2} - z|\delta)}{\partial \delta} > 0$, and $G_{x_1}(z, N) > G_{x_2}(z, N)$ for all $z \in (0, \frac{1}{2})$.

Q.E.D.

Proof of Proposition 4: We follow the same argument that we have used in the proof of Proposition 2. *Q.E.D.*

Proof of Theorem 5: The auctioneer's problem is

$$\delta^A \in \operatorname{argmax}_{\delta^A} E_{x_2}\{V(x_2, \delta) - \delta\}$$

This problem is equivalent to

$$\delta^A \in \operatorname{argmax}_{\delta^A} E_{x_1, x_2}\{V(x_1, \delta) - \delta - \Pi_w(\delta)\}$$

where $\Pi_w(\delta) = E_{x_1}\{V(x_1, \delta)\} - E_{x_2}\{V(x_2, \delta)\}$ are the expected informational rents of the winning bidder. From comparing this formulation of the auctioneer's problem to the formulation of the social welfare maximization problem (equation 1), it is clear that we cannot have $\delta^A = \delta^E$, as

the first order conditions cannot be satisfied for the same δ for the two problems. Furthermore, if $\delta^A > \delta^E$, we would have

$$E_{x_2, x_1} \{V(x_1, \delta^A) - \delta^A - \Pi_w(\delta^A)\} \geq E_{x_2, x_1} \{V(x_1, \delta^E) - \delta^E - \Pi_w(\delta^E)\}$$

As, by Proposition 3, $\Pi_w(\delta)$ is increasing this would imply

$$E_{x_2, x_1} \{V(x_1, \delta^A) - \delta^A\} > E_{x_2, x_1} \{V(x_1, \delta^E) - \delta^E\}$$

a contradiction. *Q.E.D.*

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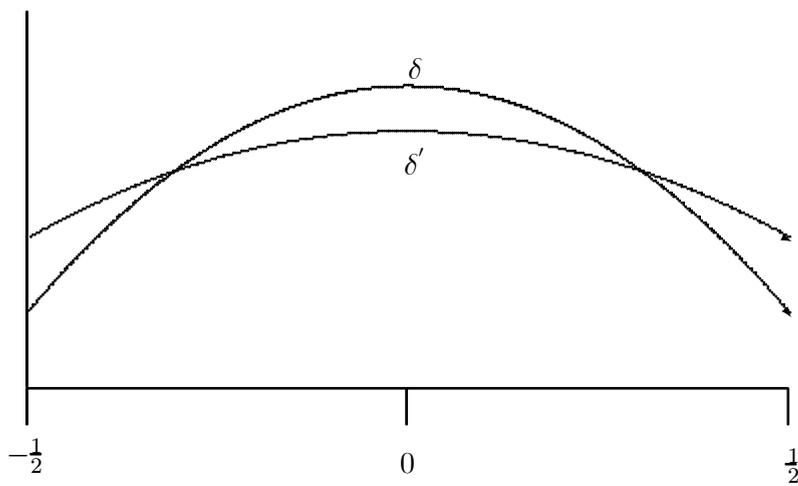


Figure 1: Two arbitrary density functions of ε where $\delta > \delta'$.