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**Personal Redistribution and the Regional
Allocation of Public Investment**

Ramon Caminal

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Ramon Caminal
Institut d'Anàlisi Econòmica, CSIC, and CEPR
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Abstract

How should an equity-motivated policy-maker allocate public capital (infrastructure) across regions. Should it aim at reducing inter-regional differences in per capita output, or at maximizing total output? Such a normative question is examined in a model where the policy-maker is exclusively concerned about personal inequality and has access to two policy instruments: (i) a personal tax-transfer system (taxation is distortionary), and (ii) the regional allocation of public investment. I show that the case for public investment as a significant instrument for interpersonal redistribution is rather weak. In the most favorable case, when the tax code is constrained to be uniform across regions, it is optimal to distort the allocation of public investment in favor of the poor regions, but only to a limited extent. If the tax code can vary across regions then the optimal policy may involve an allocation of public investment distorted in favor of the rich regions.

JEL classification numbers: D31, H41

Key words: public investment, regional policy, redistribution

1 Introduction

In most countries regional productivity differences are quantitatively significant. Governments may be tempted to reduce these differences, in particular, through the allocation of public capital. In doing so, policy-makers

may be motivated by efficiency considerations. For example, persistent regional inequalities may induce massive migration flows which may cause negative externalities to both the destination as well as the origin regions. Also, policy-makers may be concerned about equity. However, in this case it is not clear why one should take the region as the functional unit of analysis. In other words, including the reduction of regional inequality as an additional policy goal requires some justification.

Suppose policy-makers are exclusively concerned about the distribution of welfare over individuals (or households). Since regions typically have different distributions of individual characteristics the geographic allocation of public investment could still be a useful device to reduce interpersonal inequalities. However, under this approach, we must compare the costs and benefits of such an instrument to those of alternative devices; in particular, personal taxes and transfers. The goal of this paper is precisely to study from a normative point of view the potential role of the regional allocation of public capital in reducing individual welfare inequality, when a particular tax-transfer scheme is also available.

The main ingredients of the analysis are the following. First, I model public capital as an input of the aggregate production function that enhances individual productivities. In particular, I assume that an increase in the stock of public capital in a certain region multiplies the productivity of all the residents by the same factor. Thus, in absolute terms more productive individuals benefit more than less productive ones. Second, the government has access to an alternative mechanism to influence the personal welfare distribution. In particular, I consider a proportional tax on income and a uniform transfer to each individual (these policy parameters may or may not differ across regions). Taxes are assumed to be distortionary and hence the optimal policy balances the gains from redistribution and the efficiency costs. I also make various simplifying assumptions to keep the analysis tractable, like considering only two periods and two regions of equal size. The social planner is endowed with a social welfare function, which is assumed to exhibit constant elasticity of substitution over individual utilities. Hence, the intensity of the equity motive is exclusively reflected in the parameter measuring the elasticity of substitution.

I use this model to ask the following question: should the regional allocation of public investment be distorted in favor of the poor region? The answer depends crucially on the characteristics of the tax code. If tax rates and lump-sum transfers are uniform across regions then, under mild con-

ditions on the distributions of personal characteristics, the optimal policy is such that the marginal return of public investment in the rich region is higher than in the poor; that is, the allocation of public investment is distorted in favor of the poor region.¹ Thus, the allocation of public investment across regions can help the tax-transfer mechanism in improving the distribution of personal welfare. However, the contribution of public investment to individual equity can only be modest. The reason is that more egalitarian policy makers, on the one hand, are more willing to distort the regional allocation of public investment in order to reduce interregional productivity differences and achieve a more equitable income distribution; but, on the other hand, they prefer larger public transfers, since low productivity individuals only get a low direct benefit from public investment and their welfare is relatively more sensitive to public transfers. It turns out that public transfers can be increased by allocating public investment more efficiently, which expands the tax base and increases revenue.

The balance between these two effects induces a non-monotonic relationship between the distortion in the regional allocation of public investment and the intensity of the equity motive. At one extreme of the parameter space, when the social welfare function exhibits infinite elasticity of substitution, the goal is to maximize average utility (utilitarian social planner), which implies the maximization of the present value of aggregate consumption and hence an efficient allocation of public investment. At the other extreme, the social welfare function exhibits zero elasticity of substitution and hence the goal is to maximize the welfare of the least productive individuals (Rawlsian social planner). If the pre-tax income of the least productive individuals is unaffected by public investment (like in the case of retired or severely handicapped people), then the optimal allocation of public investment is also efficient. The reason is that fiscal policy only influences the utility of the least productive individuals through the lump-sum transfer. As a result, the optimal policy maximizes tax revenue, which implies again the maximization of total output. However, for intermediate cases, the optimal allocation of public capital is such that the marginal product in the rich region is higher than in the poor region (public investment is redistributed in favor of the poor region), although numerical examples suggest

¹Redistribution in favor of the poor region does not necessarily imply that the latter obtains more investment funds (relative to the size of the region). In fact, if the equity motive is sufficiently strong or sufficiently weak, then the rich region gets more funds than the poor region.

that the size of such a distortion is very small.

The distortion in the allocation of public capital can also be explained as follows. If public capital is efficiently allocated then, for a given amount of public investment, total output is maximized but personal income inequality is exacerbated (there are more highly productive individuals in the rich region, whose income is boosted by public investment). Hence, an efficient allocation of investment will require more redistribution through the tax-transfer system, which involves an efficiency cost (the dead-weight loss associated to distortionary taxation). Instead, if one unit of public capital is redistributed from the rich to the poor region, the negative effect on total output is of second order magnitude, but the associated reduction in distortionary taxation has a first order effect. In other words, in order to reach a certain level of personal redistribution it is efficient to use a combination of both distortionary taxes and a distorted allocation of public capital.

The (limited) role of public investment as a redistribution device depends crucially on the assumption of a uniform tax code across regions. If the tax code can also vary across regions then results change dramatically. In this case, policy-makers have access to a larger set of instruments for "regional redistribution". For instance, they can set a higher lump-sum transfer in the poor region. In fact, this is what happens in the example presented in Section 4: the optimal policy tends to distort the efficient allocation of public investment in favor of the rich region!. The reason is that individuals of the poor region benefit from a higher lump-sum transfer, which has to be partially compensated by a less favorable allocation of public capital.

After the seminal paper by Aschauer (1989), the literature on the effect of public capital accumulation on the productivity of private inputs has grown considerably.² Despite of some methodological difficulties there is sufficient evidence of the positive contribution of at least some categories of public capital (like infrastructures). With respect to the regional dimension most of the empirical studies have focused on the measurement of regional spillovers.³ On the equity-efficiency trade-off, De la Fuente (1996) has studied for the case of Spain the determinants of the regional allocation of public investment. The evidence suggests that policy makers have

²See, for instance, the early survey by Gramlich (1994).

³See Hulten and Schwab (1997) for a discussion and references.

deviated from the efficient allocation, and have aimed at reducing regional differences in income per capita.⁴

To the best of my knowledge, the relationship between the regional allocation of public investment and interpersonal redistribution has not been explored in a formal model. In very different frameworks Michel et al. (1983) and Takahashi (1998) have taken a normative approach and studied the optimal allocation of public spending, although they abstract from intraregional income inequality. Similarly, Persson and Tabellini (1994), Lockwood (1999) and Cheikbossian (1997) present political economy models to study the regional allocation of public spending, but again without looking at intraregional income inequality.⁵ Finally, Persson and Tabellini (1996a and b) study the design of fiscal federations as optimal risk sharing arrangements. They focus on inter-regional transfers and worry about personal redistribution, but they do not consider productive public expenditure.

In the next section I present the baseline model. The optimal policy under the constraint of identical tax codes in both regions is analyzed in Section 3. Such a constraint is relaxed in Section 4. Finally, some concluding remarks close the paper.

2 The baseline model

Consider a two-region and two-period economy. Regions are indexed by i , $i = A, B$, and periods by t , $t = 1, 2$. Each region is populated by a continuum of agents of equal mass. Individuals are heterogeneous and characterized by a parameter θ , which can be interpreted as an index of their productivity. Individual productivities are distributed in region i according to the density function $h^i(\theta)$, which takes strictly positive values in the interval $[0, \bar{\theta}]$. The cumulative distribution function is denoted by $H^i(\theta)$.

⁴See also De la Fuente and Vives (1995). Recently, De la Fuente (2001) has used a simplified version of the model presented in this paper to evaluate Spanish regional policy. He concludes that public capital has been too redistributive.

⁵In fact, Persson and Tabellini (1994b) consider two alternative models. In one of them, there are no intraregional productivity differences, and the regional allocation of public investment is determined as the outcome of a lobbying game. In the other model, individual characteristics vary within each region and fiscal policy is concerned about redistribution. However, fiscal policy consists of linear taxes and lump-sum transfer; moreover, the central government is constrained to set the same transfer in both regions.

A resident of region i , endowed with parameter θ , enjoys an income equal to θ in the first period and $\theta f(g^i)$ in the second, where g^i is the first period public investment in region i , and $f(\cdot)$ is an increasing and concave function, which also satisfies the Inada conditions. Thus, public investment increases individual productivity by the same relative amount. Hence, those agents with a higher value of θ benefit more in absolute terms. The government, as well as private agents, have access to a perfect bond market at an exogenous interest rate, r .

Such a set up can be interpreted as the reduced form of a model of a small open economy with perfect capital mobility, Walrasian markets, heterogeneous labor endowments (in efficiency units) and public capital as an argument of the aggregate production function.⁶

Let $\hat{\theta}^i$ and $\hat{\theta}$ be the average of θ in region i and in the entire economy, respectively, i.e., $\hat{\theta} = \frac{1}{2} (\hat{\theta}^A + \hat{\theta}^B)$. Regions have different average productivities. I denote region A as the relatively rich region, i.e., $\hat{\theta}^A > \hat{\theta}^B$. Also, if we let $\eta(\theta) \equiv \left(\hat{\theta}^A\right)^{-1} h^A(\theta) - \left(\hat{\theta}^B\right)^{-1} h^B(\theta)$, I assume that there exist a $\theta_0 \in (0, \bar{\theta})$ such that $\eta(\theta) < 0$ for all $\theta \in [0, \theta_0]$, and $\eta(\theta) > 0$ for all $\theta \in [\theta_0, \bar{\theta}]$. This is a sufficient condition for some of the results although it is not necessary. It literally says that the two density functions weighted by their own averages cross only once, and that there are sufficiently more low productivity individuals in region B than in region A .

Private agents derive utility only from consumption. Since the interest rate is exogenous, individual welfare depends exclusively on the present value of disposable income and hence we do not need to consider explicitly the consumer's optimization problem.

The government taxes income in both regions at the same rate τ_t . Tax revenue can be either distributed as a lump-sum transfer to individuals (independently of their region), T ,⁷ or can be used to finance public investment, $g^A + g^B$. If we denote by $u^i(\theta)$ the utility of an agent located in region i and endowed with parameter θ , and we let β be the discount

⁶Thus, the underlying aggregate production function of region i is of the following type: $Y^i = F(g^i, L^i) = f(g^i) L^i$, where L^i denotes aggregate labor in efficiency units. Considering private capital would considerably complicate the analysis unless we make very restrictive assumptions on the joint distribution of labor endowments and holdings of private capital.

⁷Alternatively, the case of region-specific tax codes is considered in Section 4.

factor, $\beta \equiv (1 + r)^{-1}$, we can write:

$$u^i(\theta) = (1 - \tau_1)\theta + T + \beta(1 - \tau_2)f(g^i)\theta \quad (1)$$

I assume (but I do not model explicitly) that taxation is distortionary. This is captured by assuming that the marginal revenue of an extra unit of income is $\tau_t - \frac{1}{2}(\tau_t)^2$, that is there is a dead-weight loss of $\frac{1}{2}(\tau_t)^2$ times the tax base.⁸ Hence, T is constrained to be non-negative. Then, the government's budget constraint can be written as:

$$\left[\tau_1 - \frac{(\tau_1)^2}{2}\right]\hat{\theta} + \beta\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^A f(g^A) + \hat{\theta}^B f(g^B)}{2} \geq T + \frac{g^A + g^B}{2} \quad (2)$$

The assumption of perfect capital markets and exogenous interest rates implies that the timing of transfers is irrelevant. Thus, T denotes the present value of transfers. However, since taxes are distortionary the timing of revenues does matter.

The government is endowed with a social welfare function with constant elasticity of substitution, σ^{-1} , $\sigma \geq 0$:

$$W = \frac{1}{1 - \sigma} \left\{ \int u^A(\theta)^{1-\sigma} dH^A(\theta) + \int u^B(\theta)^{1-\sigma} dH^B(\theta) \right\} \quad (3)$$

A value of $\sigma = 0$ implies that the government has linear preferences with respect to individual payoffs (utilitarian). In the limit as σ goes to ∞ , social welfare depends only on the lowest individual utility (Rawlsian). In general, a higher value of σ reflects a higher preference for equity.

3 Optimal policy under uniform tax codes

If taxes are non-distortionary then it is immediate to show that for any $\sigma > 0$ the optimal policy consists of $\tau_1 = \tau_2 = 1$ and $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B) =$

⁸Such an specification is very convenient and rather common in the literature (See, for instance, Bolton and Roland, 1997). All qualitative results would be identical in case we model explicitly the distortions associated to taxation. In the current formulation the maximum of the Laffer curve is reached at $\tau = 1$. Again, this is not substantial. Also, throughout the paper I assume that potential tax revenue is sufficient to finance the first best level of public investment.

β^{-1} . Thus, there is no conflict between redistribution and efficiency and policy instruments fully specialize: The tax-transfer system takes care of redistribution and public investment responds exclusively to efficiency considerations (the marginal productive of public investment is equalized across regions. However, under distortionary taxation the analysis is more complex. In particular, the government's optimization problem consists of choosing $\{\tau_1, \tau_2, T, g^A, g^B\}$ in order to maximize the objective function 3 subject to constraints 1, 2 and $T \geq 0$.

The redistribution role of public investment is captured by the variable Z , which is given by:

$$Z = \frac{\hat{\theta}^A f'(g^A)}{\hat{\theta}^B f'(g^B)}$$

Thus, depending on whether $Z = 1$, $Z > 1$, or $Z < 1$, the regional allocation of public investment is efficient, distorted in favor of the poor region, or distorted in favor of the rich region, respectively. The following proposition characterizes the optimal regional allocation of public investment.

Proposition 1 *If $\sigma = 0$, public investment is efficiently allocated across regions, i.e., $Z = 1$. For any σ , $0 < \sigma < \infty$, public investment is redistributed in favor of the poor region, i.e., $Z > 1$. Finally, in the limit as σ goes to ∞ , public investment is also efficiently allocated across regions.*

If $\sigma = 0$, fiscal policy is designed to maximize the present value of aggregate consumption. Thus, any amount of public investment must be efficiently allocated across regions. In fact, an utilitarian social planner will set $T = 0$ (no redistribution) and a constant tax rate, $\tau_1 = \tau_2$, sufficiently high to finance the optimal level of public investment. If σ goes to infinity, the social planner is exclusively concerned about the welfare of the poorest individuals, i.e., those with $\theta = 0$. Thus, since public investment has no direct effect on their welfare, fiscal policy aims at maximizing the lump-sum transfer, which implies again that public investment must be efficiently allocated (and tax rates are confiscatory, $\tau_1 = \tau_2 = 1$). However, for intermediate values of σ , public investment is used as a complementary redistribution device. The reason is that the tax and transfer system balances the gains from redistribution and the efficiency losses. As a result, a certain degree of welfare inequality remains. Moreover, if public investment

is efficiently allocated across regions then interpersonal income inequalities are exacerbated, since individuals in the rich region benefit from a higher level of public investment. If the government reallocates one unit of investment from the rich to the poor region, the loss of output is only second order but it results in a more equitable income distribution, which allows for a reduction of tax rates, which has a first order effect on efficiency.

Summarizing, the extent to which the regional allocation of public investment must be used as a redistribution device is non-monotone with respect to the intensity of the equity motive (parametrized by σ). At the same time, these results suggest that the redistribution role of public investment is rather limited. The reason is that low productivity individuals in both regions have a strong preference for an efficient distribution of public investment, because it maximizes tax revenues and hence the lump-sum transfer. A numerical example can be useful at this point.

EXAMPLE

Suppose that θ can take two values: 0 and 1. The proportion of high productivity individuals ($\theta = 1$) in region i is μ^i , i.e., $\widehat{\theta}^i = \mu^i$. Also take $f(g) = g^\lambda$. Public investment affects the relative differences in income per capita according to:

$$\frac{\widehat{\theta}^A f(g^A)}{\widehat{\theta}^B f(g^B)} = \frac{1}{Z} \left(\frac{\widehat{\theta}^A}{\widehat{\theta}^B} \right)^2$$

Figure 1 plots the optimal values of T and Z as a function of σ for the case $\lambda = 0.5$, $\mu^A = 0.7$, $\mu^B = 0.3$, $\beta = 1$. T monotonically increases with σ , i.e., there is more redistribution through the personal tax-transfer system as the intensity of the equity motive increases. In contrast, Z is non-monotonic (bell shaped and skewed to the right) and always higher than one. Hence, public investment is also used as a complementary redistribution device only for intermediate values of σ and to a very limited extent. Notice that in the optimal policy Z is at most 1.025.⁹ As a result, independently of the intensity of the equity motive, the optimal public investment policy does not significantly reduce relative interregional differences in income per capita. In case of an efficient allocation of public investment the ratio of (second period) average income in regions A and B is 5.43, and in the optimal policy such a ratio is never reduced by more than one per cent.

⁹Results are similar in all examples considered. See Caminal (2000) for details.

4 Optimal policy under region-specific tax codes

Suppose that the government can set different tax rates and transfers in the two regions. Thus, the set of instruments is now: $\{\tau_t^i, T^i, g^i\}$, $t = 1, 2$ and $i = A, B$. The analogous of equations 1 and 2 are:

$$u^i(\theta) = (1 - \tau_1^i)\theta + T^i + \beta(1 - \tau_2^i)f(g^i)\theta \quad (4)$$

$$\sum_{i=A,B} \left\{ \left[\tau_1^i - \frac{(\tau_1^i)^2}{2} \right] \hat{\theta}^i + \beta \left[\tau_2^i - \frac{(\tau_2^i)^2}{2} \right] \hat{\theta}^i f(g^i) \right\} \geq \sum_{i=A,B} (T^i + g^i) \quad (5)$$

The government's optimization problem consists of choosing $\{\tau_t^i, T^i, g^i\}$ in order to maximize the objective function 3 subject to constraints 4, 5, and $T^i \geq 0$. It turns out that, as in the previous section, the regional allocation of public investment is efficient for extreme values of σ :

Proposition 2 *If $\sigma = 0$ and in the limit as σ goes to ∞ , public investment is efficiently allocated across regions, i.e., $Z = 1$.*

The intuition is similar to the one in the previous section. If $\sigma = 0$, the social planner is only concerned about efficiency and hence the regional allocation of public investment must also be efficient. In fact, an utilitarian social planner will set $T^A = T^B = 0$ (no redistribution) and $\tau^A = \tau^B$, in order to minimize the dead-weight loss of financing investment. As σ goes to infinity the social planner only cares about the utility of the poorest individuals, and hence tax revenue should be maximized, which implies that $\tau^A = \tau^B = 1$, and that public investment must be efficiently allocated across regions. Finally, transfers must be equal across regions: $T^A = T^B$.

Proposition 2 is silent about the regional allocation of public investment for intermediate values of σ . In fact, by means of an example, it can be shown that in the optimal policy Z can be higher or lower than one. Let us consider again the example of Section 3. Figure 2 plots the value of Z , $\tau^A - \tau^B$, $T^B - T^A$, as a function of σ , for the same parameter values of Figure 1. Except for very small values of σ , it turns out that $Z < 1$, that is public investment in general must be redistributed in favor of the rich region. The reason has to do with the fact that "regional redistribution" is

essentially conducted through $(T^B - T^A)$, which notice that is positive for all values of σ .¹⁰

In other words, the main message of this section is that whenever the tax code can vary across regions it is not clear whether the social planner uses public investment as a redistribution device. It may be the case that in the optimal policy public investment is distorted in favor of the rich region to partially compensate the differential in the lump-sum transfers. In fact, for the above specification of the production function and income distributions, and for all the parameter values considered,¹¹ the same pattern of Figure 2 is observed, i.e., as long as σ is not too small, then $Z < 1$.

5 Concluding remarks

The analysis has been conducted in a highly simplified framework. The working paper version (Caminal, 2000) discusses in some detail how results are modified when we relax some assumptions. First, if fiscal policy has a positive but small effect on the pre-tax income of the least productive individuals then a Rawlsian social planner also distorts the allocation of public investment in favor of the poor region although such a distortion is also small. Second, if the group of individuals with the lowest productivity resides in the region with higher average productivity then the optimal policy may involve distorting the efficient allocation of public capital in favor of the rich region.

However, it is difficult to generalize some other important assumptions. For instance, considering non-linear tax-transfer schemes requires an explicit analysis of the distortions associated with taxation. Nevertheless, one may conjecture that under more progressive taxation the poor are able to appropriate a higher proportion of the returns from public investment and thus the arguments in favor of an efficient allocation of public investment are enhanced.

Another important assumption was the linearity of the public investment-productivity technology. I am not aware of any empirical evidence in favor or against it. Given the lack of information linearity seems a natural start-

¹⁰In fact, in the case that the transfer T can vary across regions but not the tax rate, $\tau_t^A = \tau_t^B$, it is also possible to find parameter values for which in the optimal policy $Z < 1$ and others for which $Z > 1$.

¹¹See Caminal (2000) for details.

ing point. However, I find it plausible that some types of public investment increase the productivity of individuals at the upper tail of the distribution more than proportionally, while others may increase it less than proportionally. Thus, in principle, if governments choose a different composition of public investment for different regions, they might have a better chance of addressing distributional objectives than in the current model.¹²

Other hypotheses of the paper deserve further attention. On the one hand, I have assumed that individuals do not move across regions. As more people are more willing to move for a certain regional fiscal gap, it becomes more difficult for the government to discriminate across regions, either through the tax code or through the allocation of public investment. Also, it becomes crucial how economic and non-economic incentives interact in migrants' decisions. Will the poor or the rich be more willing to move if by doing so they obtain the same proportional increase in their disposable income? On the other hand, I have assumed that there are no spillover effects. Although most investment projects have a large local effect, in most cases their benefits reach citizens in other regions (and other countries). This is an important consideration which is at the core of the literature on fiscal federalism.¹³ Clearly, it would also affect the policy design issue I have been discussing in this paper. With asymmetric regions, spillovers from public investment are also likely to be asymmetric. Consequently, the problem of allocating public investment across regions may drastically change depending on the size and direction of such asymmetric spillover effects. These two issues are left for future research.

¹²Thus, the results of this paper apply to the case that governments only control the regional allocation of "aggregate" public investment, perhaps because they ignore the distributional consequences of each specific type of public investment..

¹³See, again, Hulten and Schwab (1977).

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7 Appendix

7.1 Proof of Proposition 1

If we denote by δ the Lagrange multiplier associated with the constraint $T \geq 0$, the first order conditions are:

$$\frac{dW}{d\tau_1} = \sum_{i=A,B} \left\{ \int u^i(\theta)^{-\sigma} [-\theta + (1 - \tau_1)\hat{\theta}] dH^i(\theta) \right\} = -\delta(1 - \tau_1)\hat{\theta}$$

$$\frac{dW}{d\tau_2} = \sum_{i=A,B} \left\{ \int u^i(\theta)^{-\sigma} [-\theta f(g^i) + (1 - \tau_2)G] dH^i(\theta) \right\} = -\delta(1 - \tau_2)G$$

where G is given by:

$$G \equiv \frac{\hat{\theta}^A f(g^A) + \hat{\theta}^B f(g^B)}{2}$$

Also for $i, j = A, B, i \neq j$:

$$\frac{dW}{dg^i} = \int u^i(\theta)^{-\sigma} [\beta(1 - \tau_2)\theta f'(g^i) + \Delta] dH^i(\theta) + \int u^j(\theta)^{-\sigma} \Delta H^j(\theta) = -\delta\Delta$$

where

$$\Delta \equiv \beta \left[\tau_2 - \frac{(\tau_2)^2}{2} \right] \frac{\hat{\theta}^i f'(g^i)}{2} - \frac{1}{2}$$

Consider first the case $\sigma = 0$. The first order conditions can be rewritten:

$$-2\tau_1 + \delta(1 - \tau_1) = 0$$

$$\tau_1 = \tau_2 \equiv \tau$$

$$\beta \hat{\theta}^i f'(g^i) \left[1 - \tau + \frac{\tau^2}{2} \right] = 1$$

Moreover, the non-negativity constraint is binding. Suppose not, i.e., $\delta = 0$. In this case $\tau = 0$, but since g^i are positive, the government budget constraint is violated. Hence, $T = 0$. Consider now the limit as σ goes to infinity. The first order condition with respect to τ_1 can be written as:

$$\int_0^{(1-\tau_1)\hat{\theta}} \Omega^A(\theta) \Gamma(\theta) dH^A(\theta) + \int_{(1-\tau_1)\hat{\theta}}^{\bar{\theta}} \Omega^A(\theta) \Gamma(\theta) dH^A(\theta) +$$

$$\left\{ \frac{u^B[(1-\tau_1)\hat{\theta}]}{u^A[(1-\tau_1)\hat{\theta}]} \right\}^{-\sigma} \left\{ \int_0^{(1-\tau_1)\hat{\theta}} \Omega^B(\theta) \Gamma(\theta) dH^B(\theta) + \int_{(1-\tau_1)\hat{\theta}}^{\bar{\theta}} \Omega^B(\theta) \Gamma(\theta) dH^B(\theta) \right\} =$$

$$= -\frac{\delta(1-\tau_1)\hat{\theta}}{\left\{ u^A[(1-\tau_1)\hat{\theta}] \right\}^{-\sigma}}$$

where:

$$\Omega^i(\theta) \equiv \left\{ \frac{u^i(\theta)}{u^i[(1-\tau_1)\hat{\theta}]} \right\}^{-\sigma}$$

$$\Gamma(\theta) \equiv -\theta + (1-\tau_1)\hat{\theta}$$

As σ goes to infinity, if $\tau_1 < 1$, the first and third integral of the left hand side go to infinity, and the second and the fourth go to zero. Also the factor multiplying the third and fourth integrals goes either to infinity or to zero. Thus, the left hand side is positive and equation does not hold. Hence, $\tau_1 = 1$. The argument to show that $\tau_2 = 1$ is analogous.

Suppose $\delta = 0$. The first order condition with respect to g^i imply that:

$$\hat{\theta}^i f'(g^i) = \frac{2}{\beta}$$

The government budget constraint becomes:

$$T = \frac{1}{2} \left[\hat{\theta} + \beta \frac{\hat{\theta}^A f(g^A) + \hat{\theta}^B f(g^B)}{2} \right] - \frac{g^A + g^B}{2} > 0$$

Finally, let us consider the case $0 < \sigma < \infty$. By direct inspection of the first order conditions τ_1 and τ_2 are strictly less than 1. Next, I write the first order condition with respect to g^i :

$$2\beta (1 - \tau_2) \widehat{\theta}^i f(g^i) \Psi^i - \left\{ 1 - \beta \left[\tau_2 - \frac{(\tau_2)^2}{2} \right] \widehat{\theta}^i f(g^i) \right\} \Pi = 0$$

where

$$\Psi^i \equiv \int u^i(\theta)^{-\sigma} \frac{\theta}{\widehat{\theta}^i} dH^i(\theta)$$

$$\Pi \equiv \int u^A(\theta)^{-\sigma} dH^A(\theta) + \int u^B(\theta)^{-\sigma} dH^B(\theta) + \delta$$

Notice that $\widehat{\theta}^A f'(g^A)$ is higher, equal or lower than $\widehat{\theta}^B f'(g^B)$ if and only if Ψ^A is lower, equal or higher than Ψ^B , respectively. Suppose that $\widehat{\theta}^A f'(g^A) \leq \widehat{\theta}^B f'(g^B)$. Then $g^A > g^B$ and $u^A(\theta)^{-\sigma} < u^B(\theta)^{-\sigma}$ for all θ . Thus,

$$\Psi^A < \int u^B(\theta)^{-\sigma} \frac{\theta}{\widehat{\theta}^A} dH^A(\theta) < \Psi^B$$

The last inequality comes from the fact that $u^B(\theta)^{-\sigma}$ is strictly decreasing and from the assumption on the single crossing property of the two density functions. Thus, we reach a contradiction and hence conclude that $\widehat{\theta}^A f'(g^A) \geq \widehat{\theta}^B f'(g^B)$. QED

7.2 Proof of Proposition 2

If we denote by δ^i the Lagrange multiplier associated with the constraint $T^i \geq 0$, the first order conditions are:

$$\tau_1^i = \tau_2^i \equiv \tau^i$$

$$\int u^i(\theta)^{-\sigma} \left[-\theta + (1 - \tau^i) \widehat{\theta}^i \right] dH^i(\theta) = -\delta^i (1 - \tau^i) \widehat{\theta}^i$$

$$\int u^A(\theta)^{-\sigma} dH^A(\theta) + \delta^A = \int u^B(\theta)^{-\sigma} dH^B(\theta) + \delta^B$$

$$\beta \widehat{\theta}^i f'(g^i) \left[1 - \tau^i + \frac{(\tau^i)^2}{2} \right] = 1$$

If $\sigma = 0$ and in the limit as σ goes to infinity we can follow the argument used in the proof of Proposition 1 and conclude that the regional allocation of public investment must be efficient. QED

Figure 1

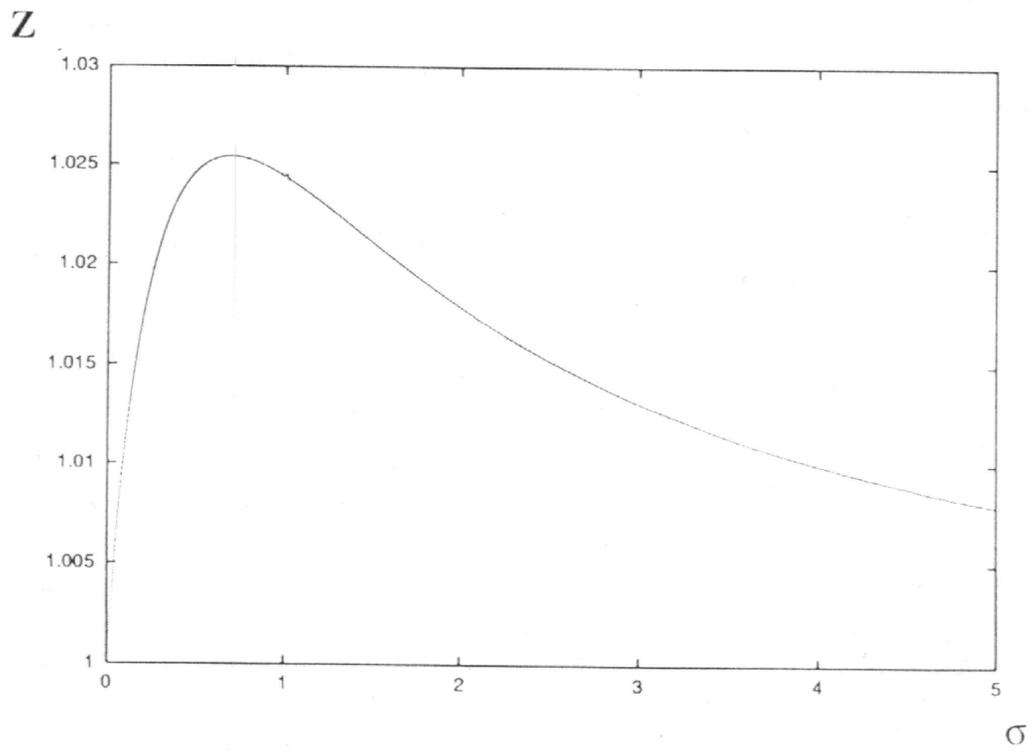
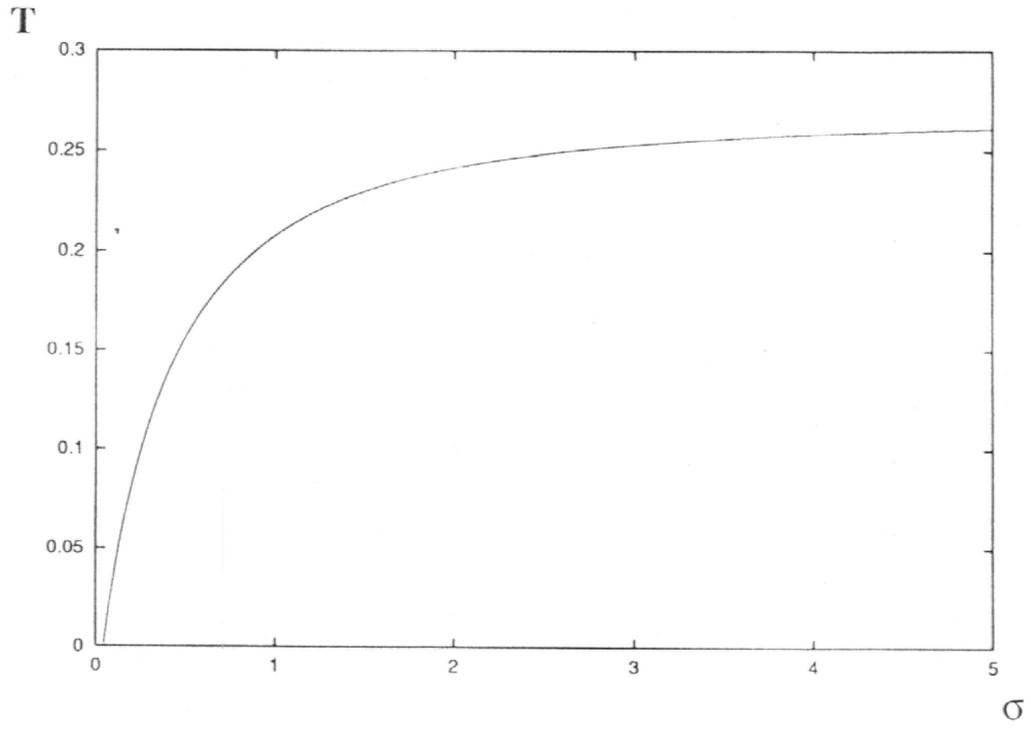


Figure 2a

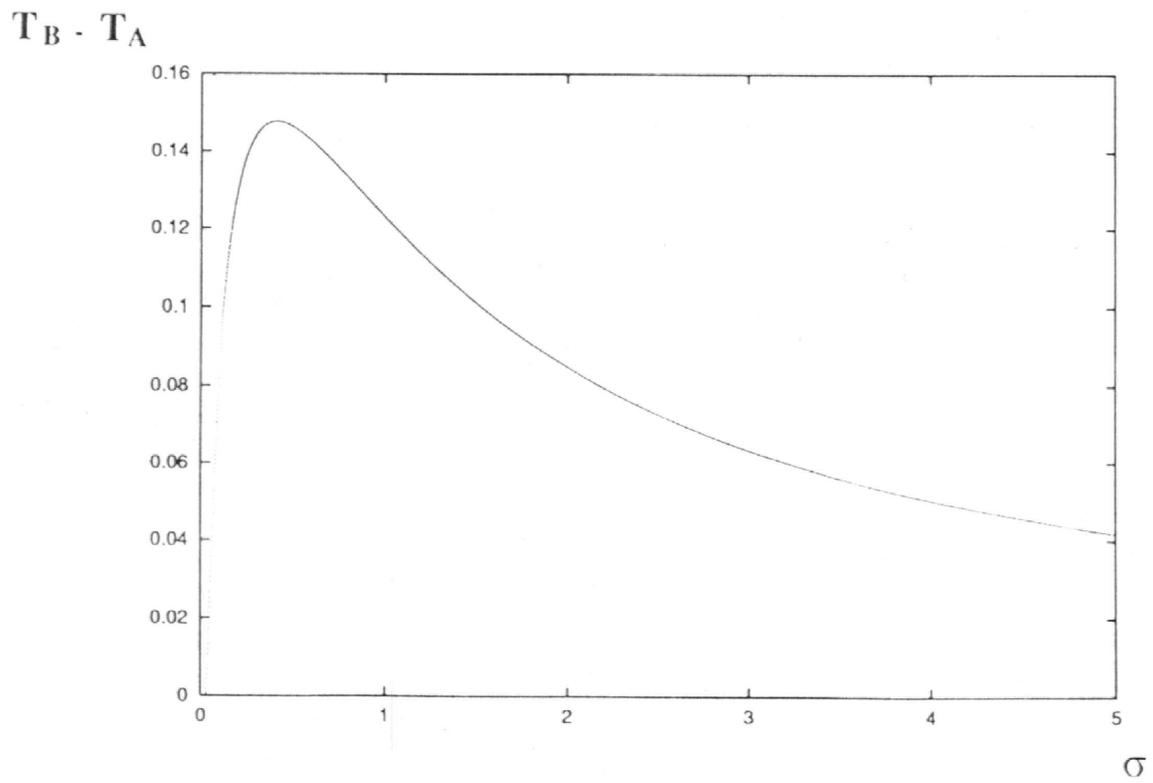
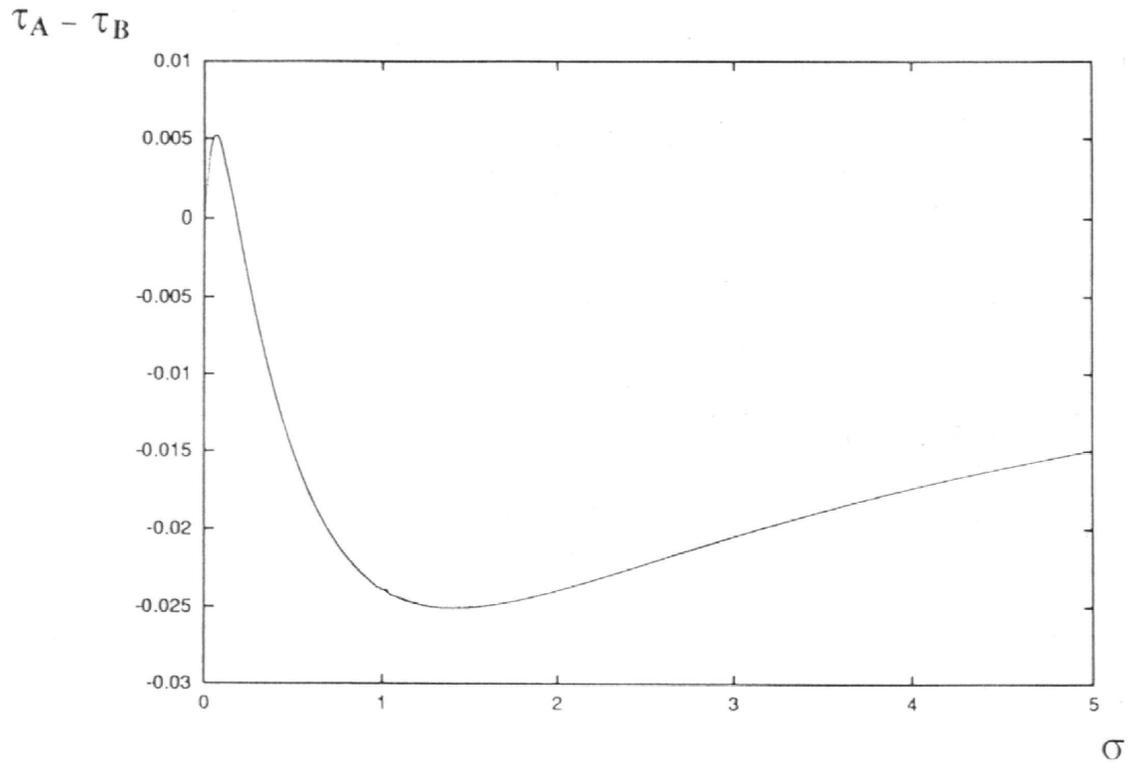


Figure 2b

